Enzo-E/Cello astrophysics and cosmology Adaptive mesh refinement astrophysics using Charm++

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Scientific questions in astrophysics and cosmology



[John Wise]

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Simulations require modelling multiple physics phenomena

Physics Equations: mathematical models

• Gravity $(\nabla^2 \Phi = 4\pi G\rho)$ • Hydrodynamics (Euler equations) • Chemistry/cooling • MHD • Cosmological expansion . . .

Linear solvers (Krylov subspace, multigrid, composite) • modified PPM • Grackle chemistry/cooling • VL+CT MHD . . .

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Numerical methods are required for solving the physics equations

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Numerical Methods	approximate and solve	Enzo-E
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Parallel methods are enabled by distributed data structures

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\bullet Linear solvers (Krylov subspace, multigrid, composite) \bullet modified PPM \bullet Grackle chemistry/cooling \bullet VL+CT MHD				
Data Structures	computer representation	Cello		

Parallel Runtime System distribute data and computation Charm++

dynamic task scheduling
 data-driven execution
 asynchronous

Charm++ 2020

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Charm++ provides the support for running on large-scale HPC platforms

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Data Structures	computer representation	Cello

• Adaptive mesh refinement (array-of-octrees) • Eulerian fields • Lagrangian particles

Parallel Runtime System distribute data and computation Ch					narn	n++
dynamic task scheduling • data-driven execution • asynchronous						
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Cosmological simulations with Enzo-E/Cello



- array-of-octrees
- blocks of data
- chare array

dark matter

- particle data
- collisionless
- CIC gravity

baryonic matter

field data

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- PPM hydro
- flux-correction

Cosmological simulations with Enzo-E/Cello



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Enzo-E/Cello astrophysics and cosmology

Cosmological simulations with Enzo-E/Cello



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baryonic matter



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- PPM hydro
- flux-correction

Field and particle data communication

Field data exchange

- send Field face data when available
- count face data messages received
- last receive triggers computation
 - Particle migration
 - scatter across 4³ pointer array
 - send to associated neighbors
 - gather incoming particles

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Particle migration

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Recent work has focused on scalable linear solvers

- Krylov subspace methods
 - CG (symmetric), BiCG-STAB (nonsymmetric)
 - easy to implement (basic linear algebra)
 - poor algorithmic scalability w/o preconditioning
 - communication intensive
- Multigrid methods
 - MG V-cycle
 - harder to implement (involves coarse blocks)
 - better algorithmic scalability
 - method limited to uniform meshes
- Composite methods
 - HG (Reynolds): multigrid-preconditioned Krylov
 - DD (Norman): domain-decomposition

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Issue #1: scalable gravity The Enzo-E domain decomposition solver DD

- 1. EnzoSolverMg0 for root-level solve
 - demonstrated good parallel scalability
 - tested to $N_0 = 2048^3$ on P = 131K BW fp-cores
- 2. EnzoSolverBiCgStab for "tree-solves"
 - use root-level solution for boundary conditions
 - no communication between root block domains
- 3. EnzoSolverJacobi for smoothing
 - smooths discontinuities across domain boundaries
 - previously available as multigrid smoother

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The Enzo-E domain decomposition solver DD



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The Enzo-E domain decomposition solver DD





- demonstrated good parallel scalability
- tested to $N_0 = 2048^3$ on P = 131K BW fp-cores



- 2. EnzoSolverBiCgStab for "tree-solves"
 - use root-level solution for boundary conditions
 - no communication between root block domains



- 3. EnzoSolverJacobi for smoothing
 - smooths discontinuities across domain boundaries
 - previously available as multigrid smoother

One cycle of a DM-only cosmology simulation using DD



J.Bordner, M.L.Norman

Enzo-E/Cello astrophysics and cosmology

Charm++ 2020

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lssue #2: robust refresh

Implementing DD Solver uncovered communication issues

Previous Refresh

send face data when available

- copy data to ghosts when received
- race condition: may not be ready!
- extra synchronization added for correctness

Current Refresh

- sends face data when available
- 1. receives but not ready \Rightarrow copy data to buffer

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- 2. becomes ready \Rightarrow copy buffers to ghosts
- 3. receives and ready ⇒ copy data to ghosts

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AMR cosmology simulations with DM + gas

- Enzo-E can run DM-only AMR simulations
- and DM + gas non-AMR simulations
- but DM + gas with AMR leads to non-physical behavior

clue: always at refinement-level boundaries

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clue: always at refinement-level boundaries

J.Bordner, M.L.Norman

Conserved quantities were not conserved at refinement jumps

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Image: A math a math

Conserved quantities were not conserved at refinement jumps



- consider two fine blocks adjacent to a coarse block
- hydro computations depend on ghost values
- ghost values available from previous refresh
- fluxes at both fine and coarse faces
- conservation requires consistent fluxes
- not consistent in general
- apply a "flux-correction" step
- update coarse values along face so fluxes match
- requires coarse-to-fine block communication

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Implementing flux-correction didn't fix the problem :-(

Ran experiments to narrow down the problem

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Implementing flux-correction didn't fix the problem :-(

Ran experiments to narrow down the problem



- problem only occurs when including gas dynamics
- internal energies blow up at level interfaces

linear interpolation was main suspect

- got further with injection but grid effects
- suspected mismatched time-centering
- using $\alpha = 0.0$ instead of $\alpha = 0.5$ didn't help
- reducing order of accuracy only delayed problem

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2nd order Laplacian and accelerations

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2nd order Laplacian and accelerations

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reducing order of accuracy only delayed problem
 2nd order Laplacian and accelerations

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Ran experiments to narrow down the problem



Issue #4: improved interpolation Lately have been implementing ENZO's interpolation scheme

ideally accurate, monotonic, conservative
 linear interpolation: coarse block values only
 some ghost cells must be extrapolated
 non-monotonic: negative densities, etc.

- ENZO's SecondOrderA method
- uses an extra layer of coarse zones
- overlaps some other adjacent blocks
- additional communication required

Lately have been implementing ENZO's interpolation scheme



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Image: A matrix and a matrix

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Issue #4: improved interpolation New interpolation scheme involves additional blocks

consider interpolating ghost cells B_i → B_j
coarse array overlaps additional blocks B_k
B_k needs to know it participates in B_i → B_j
can take advantage of symmetry
assume fully-balanced octree
B_k must be same level as B_i or B_j
if same level as B_i, then B_k → B_j
if same level as B_j, then B_i → B_k

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- if same level as B_i , then $B_k o B_j$
- if same level as B_j , then $B_i \rightarrow B_k$

Indexing gets complicated and error-prone: introduced Box class

define blocks

- size (*nx*, *ny*)
- ghosts (gx, gy)

define neighbor

- level L
- face (*fx*, *fy*)
- child (cx, cy)
- variable ghost depths to refresh
- optional ghost depths on send (e.g. mass deposit to total density)
- extra coarse-zone padding (*px*, *py*) for ENZO interpolation
- easily test block B_k intersection with defined region

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Indexing gets complicated and error-prone: introduced Box class

- define blocks
 - size (*nx*, *ny*)
 - ghosts (gx, gy)
- define neighbor
 - level L
 - face (fx, fy)
 - child (cx, cy)



- variable ghost depths to refresh
- optional ghost depths on send (e.g. mass deposit to total density)
- extra coarse-zone padding (px, py) for ENZO interpolation
- easily test block B_k intersection with defined region

Conclusions

Finishing up last steps before production runs

- 1. scalable gravity
- 2. buffered refresh
- 3. flux-correction
- 4. improved interpolation
- Just a couple remaining loose ends
 - scalable I/O
 - scalable checkpoint/restart (thanks Ronak!)
- Performance optimization required
 - Enzo-E much slower than ENZO
 - over-refinement relative to ENZO
 - refresh across edges/corners
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