Moving-Mesh Hydrodynamics in ChaNGa

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Charm++ 2020 Workshop

Outline

- Numerical Simulations of Astrophysical Phenomena
  - Eulerian, SPH, ALE – pros and cons
  - MANGA - Built on top of the SPH code ChaNGa
- Common Envelope Evolution
- Tidal Disruption Events
- General Relativistic Hydrodynamics on a Moving-mesh
- Conclusions

Results of this work appear or will appear in
Prust & Chang (2019), Prust (2020), Chang, Davis, & Jiang (2020), Chang & Etienne (2020), Spaulding & Chang (submitted)

Euler Equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \boldsymbol{v} = 0$$

continuity

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} + \nabla \cdot \rho \boldsymbol{v}\boldsymbol{v} = \nabla P - \rho \nabla \Phi$$

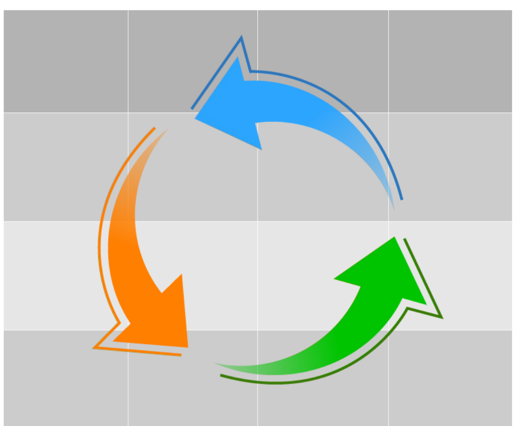
Momentum

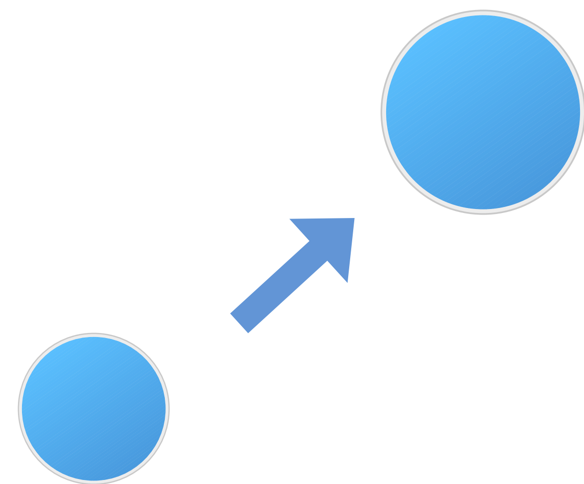
Two views of these equations

Eulerian

$$\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla = \frac{d}{dt}$$

Lagrangian (SPH)

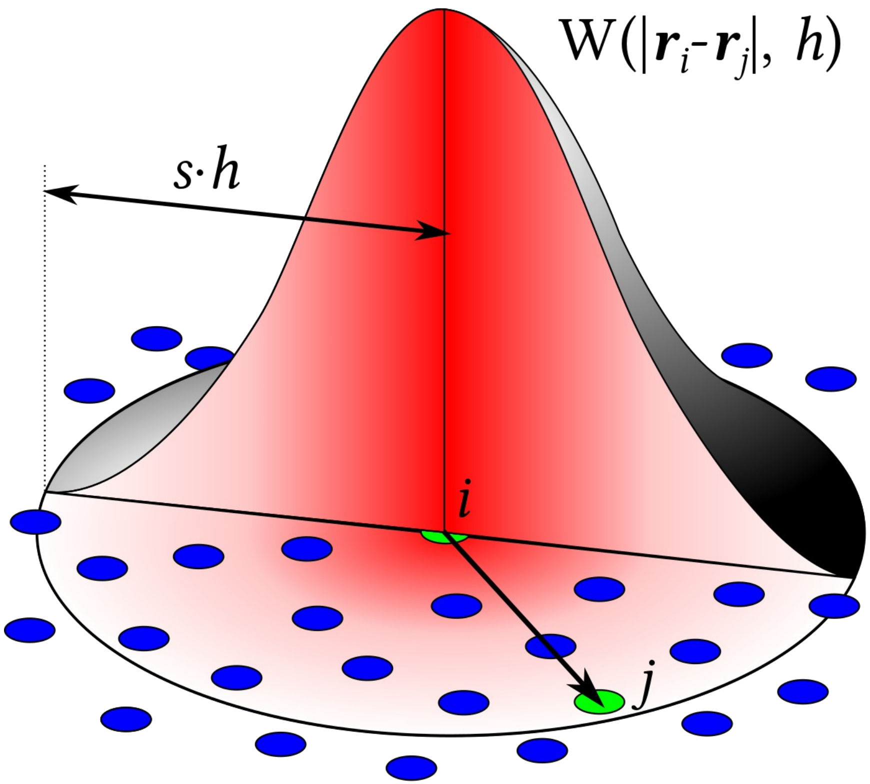




Track the fluid flow

Follow the fluid element

Smooth Particle Hydrodynamics



Wikipedia

- Model fluids as a number of discrete particles subject to F=ma forcing.
- Pressure forces depend of continuum values (density) so need an estimate for density.
- Density estimate provide by a weighed count (kernel) over a volume that includes the n-th nearest neighbors.

$$\rho_i = \frac{3}{4\pi h^3} \sum_j m_j W(|r_i - r_j|, h)$$

- Main computational challenge is doing a rapid search for the n-th nearest neighbors
- Maps well with n-body tree codes.

Eulerian Scheme

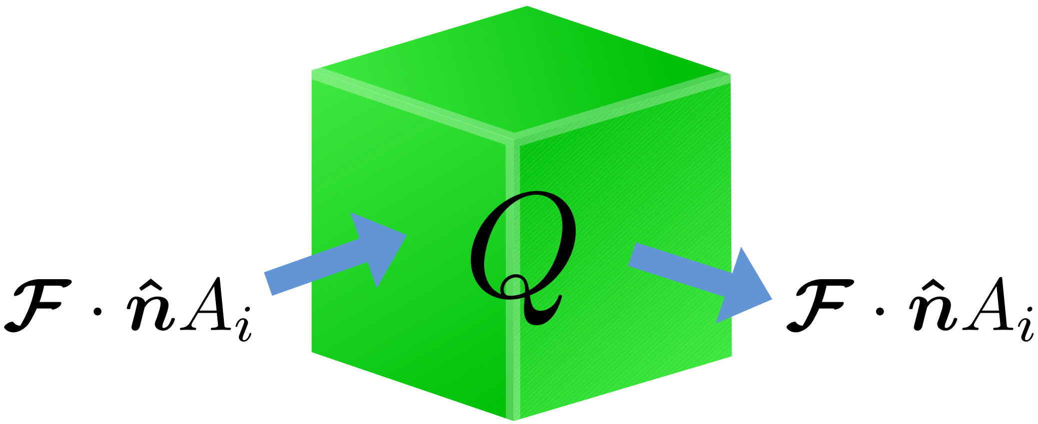
Euler equation among others can be written as a flux-conservative equation

$$\frac{\partial u}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{\mathcal{F}} = \mathcal{S}$$

$$u = (\rho, \rho \boldsymbol{v}),\ \boldsymbol{\mathcal{F}} = (\rho \boldsymbol{v}, \rho \boldsymbol{v}\boldsymbol{v} + P\mathcal{I}), \mathcal{S} = (0, -\rho \boldsymbol{\nabla}\Phi)$$

Can be solve in a finite volume scheme

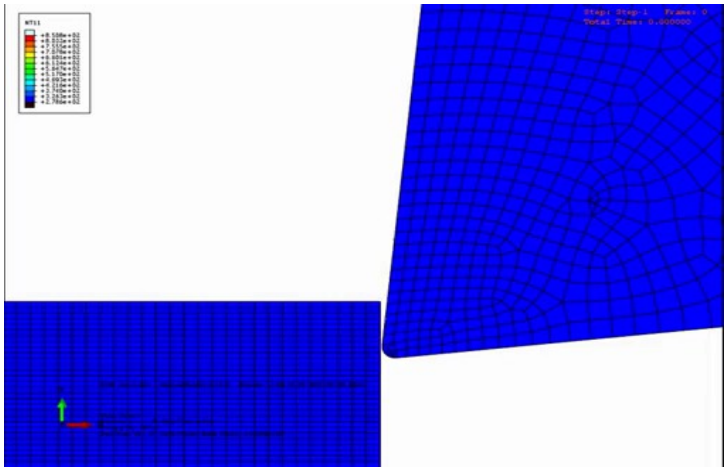
$$Q = \int u dV \qquad \int \boldsymbol{\nabla} \cdot \boldsymbol{\mathcal{F}} dV = -\int \boldsymbol{\mathcal{F}} \cdot \hat{\boldsymbol{n}} dA$$



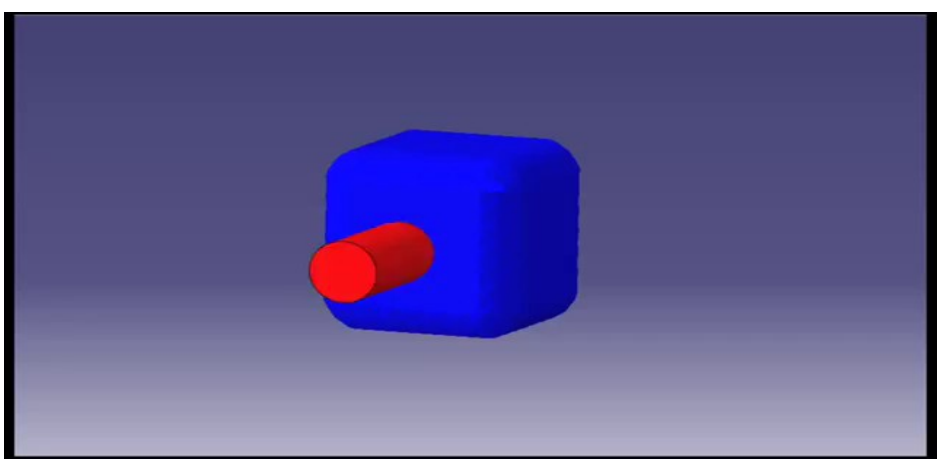
Fluxes are solved with a (approximate) Riemann solver

Arbitrary Lagrangian-Eulerian (ALE) Scheme

Abaqus finite element

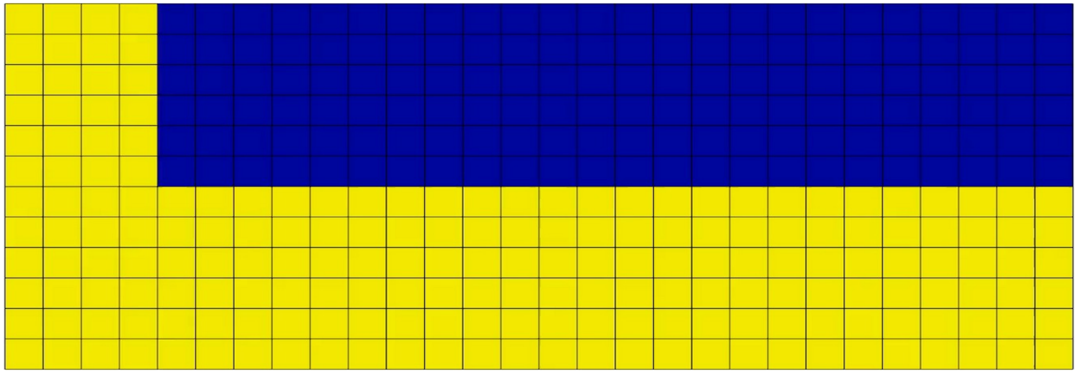


Abaqus finite element



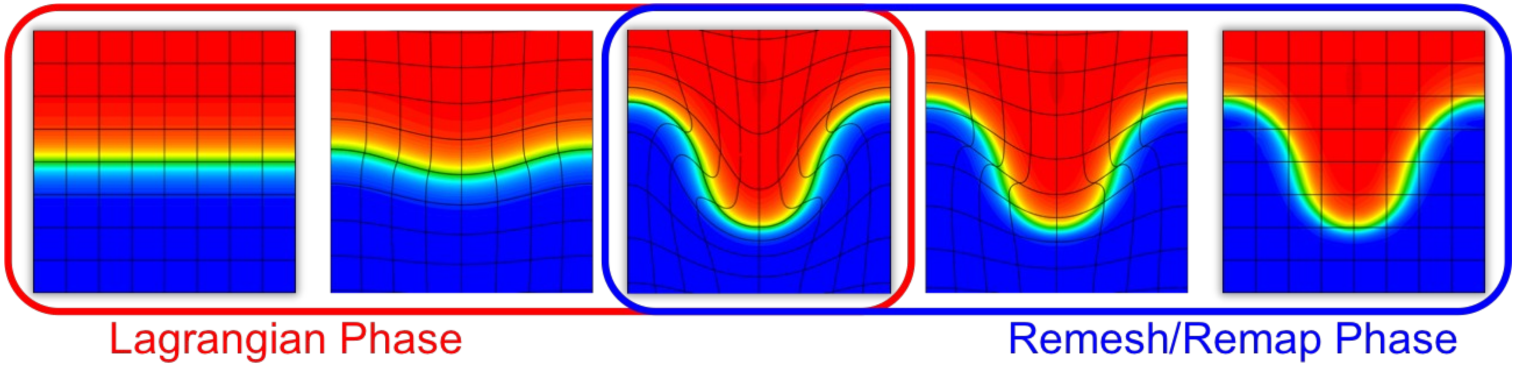
- Move the mesh cells arbitrarily
  - Usually at the local “flow” velocity
- Used in continuum mechanics
  - Meshes are unstructured
  - Strange arbitrarily shaped boundaries
- Great for fluid/solid interactions
- Big speed improvements possible if flow velocity >> sound speed

Arbitrary Lagrangian-Eulerian (ALE) Scheme



- Traditional ALE methods suffer from mesh-distortion.
- Usually requires a re-mesh – fundamentally a numerically diffusive action.
- Standard practice in continuum mechanics.

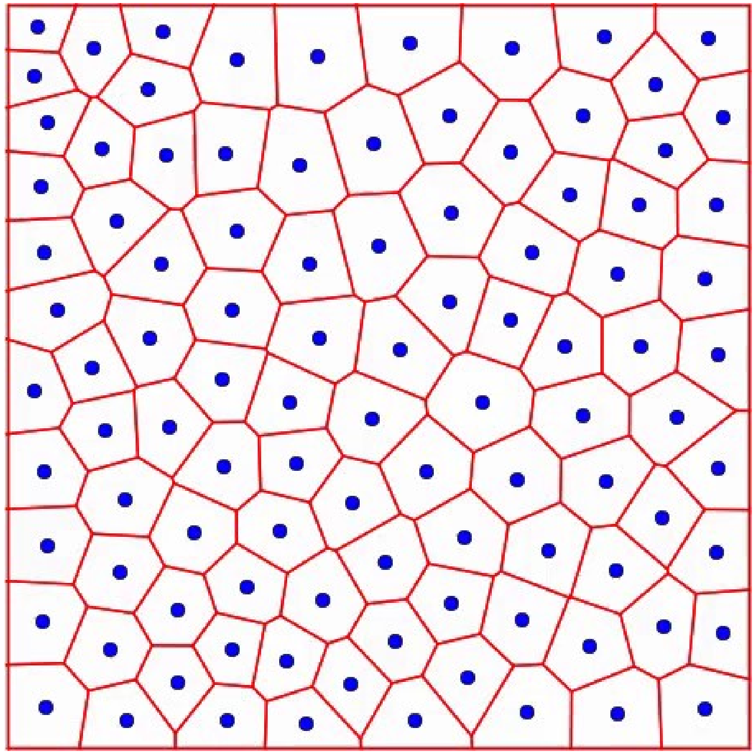
Anderson et al. 2018



- Not widely use in astronomy until about 2010.
- Development of numerical hydrodynamics on Voronoi meshes solves the problem of remeshing (Springel 2010)

Voronoi Tessellation

Vandenbroucke & De Rijcke (2016)



- Voronoi tesslation divides up space given an arbitrary distribution of points.
- Each face (edge) is a perpendicular bisecting-plane (bisector) of the line connecting adjacent points.
- Three important properties
  - Uniqueness
  - Cells are convex
  - Cells deform continuously under small perturbations.
- Well defined faces and volumes allow finite volume methods to be applied (Springel 2010).
- Any Flux-conservative equation can be solved on these unstructured meshes.
- Codes that use this methodology include AREPO (Springel 2010), RICH (Steinberg et al. 2016), TESS (Duffell & Macfadyen 2012), & MANGA (Chang et al. 2017)

Pros and Cons of Voronoi Hydrodynamics

Pros

- Far better advection than Eulerian.
- Superior conservation of momentum and angular momentum compared to Eulerian schemes
- Superior shock capturing compared to SPH.
- Better capture of interface instabilities in principle.
- Can do MHD – unlike SPH
- Continuously varying resolution – no factor of 2 or 4 jumps as in AMR.
- Almost anything solvable on Eulerian grids map to Voronoi methods.

Cons

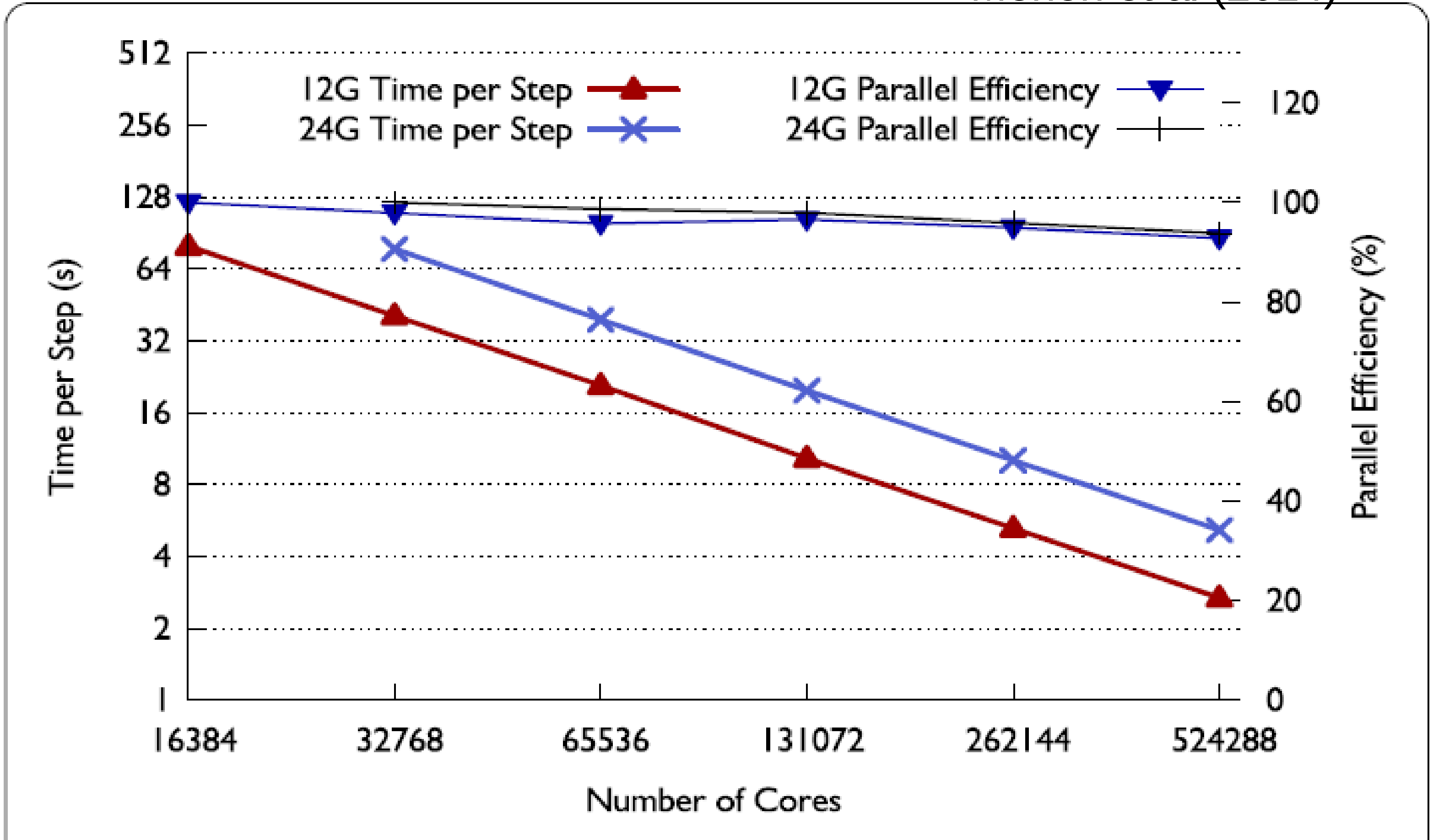
- Much more complex – combination of SPH and Eulerian + computational geometry
- Have to think about the grid (on top of everything else).
- “slower”
- MHD is divergence cleaning or vector potential based – no “staggered” CT scheme exists.
- Might be overkill for many problems

Advantages in advection, shock capturing and conservation law make it great for dynamical stellar problems.

MANGA

- Voronoi hydro solver for the Charm++ N-body Gravity (ChaNGa) – an N-body/SPH code
- Uses Charm++ programming model – “easier” to make large hybrid MPI/OpenMP codes
- ChaNGa scales in pure Gravity to 0.5M cores with 93% efficiency

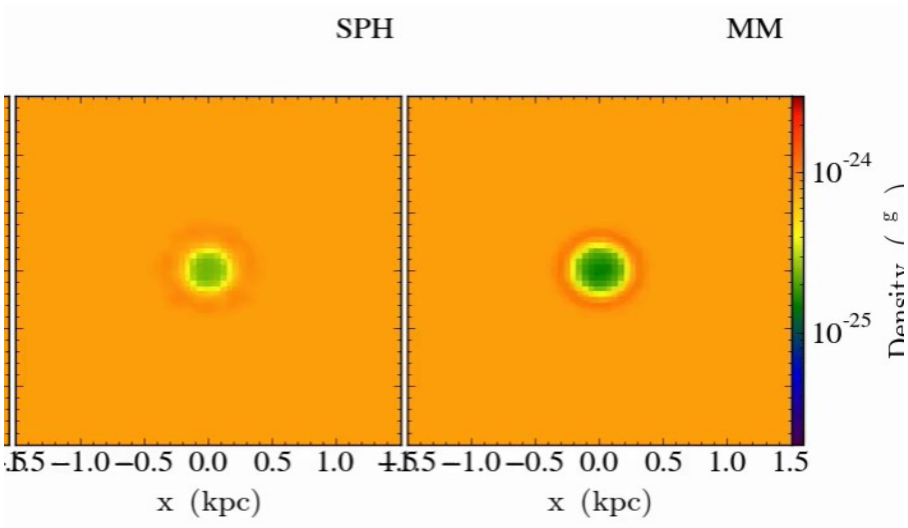
Menon et al (2014)



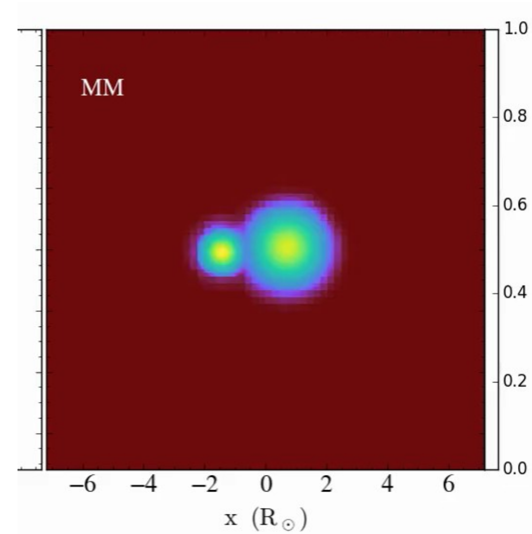
MANGA

Chang et al (2017)

Chang et al (2017)

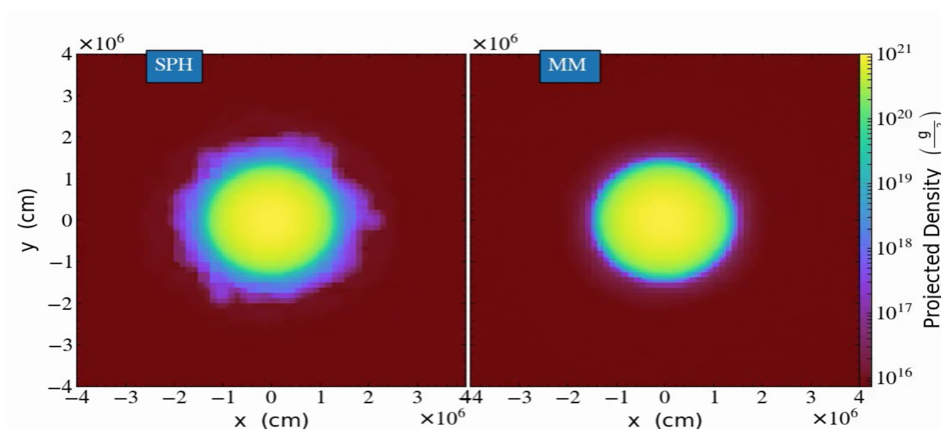


hydrodynamics



Stellar EOS

Self Gravity

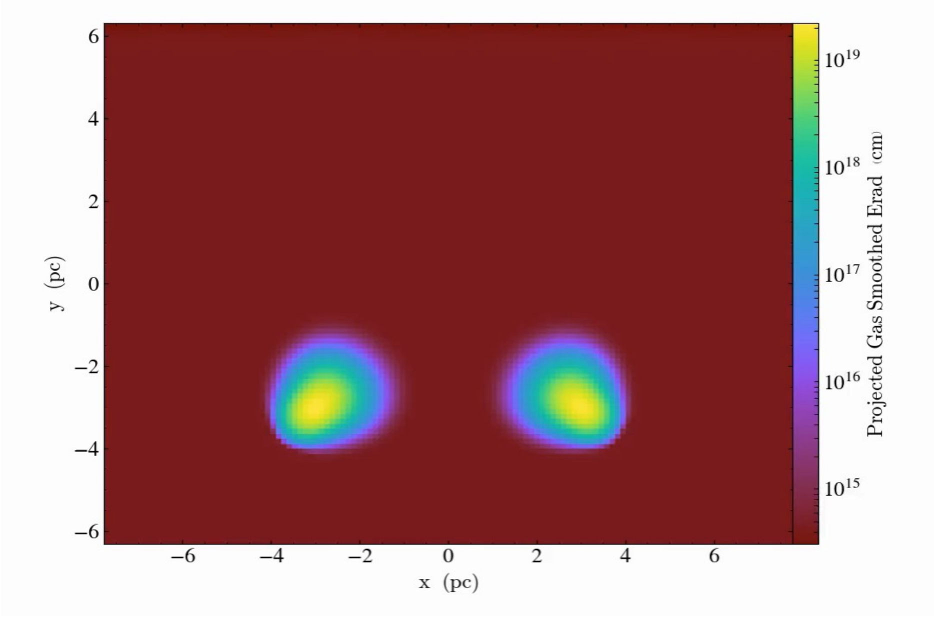


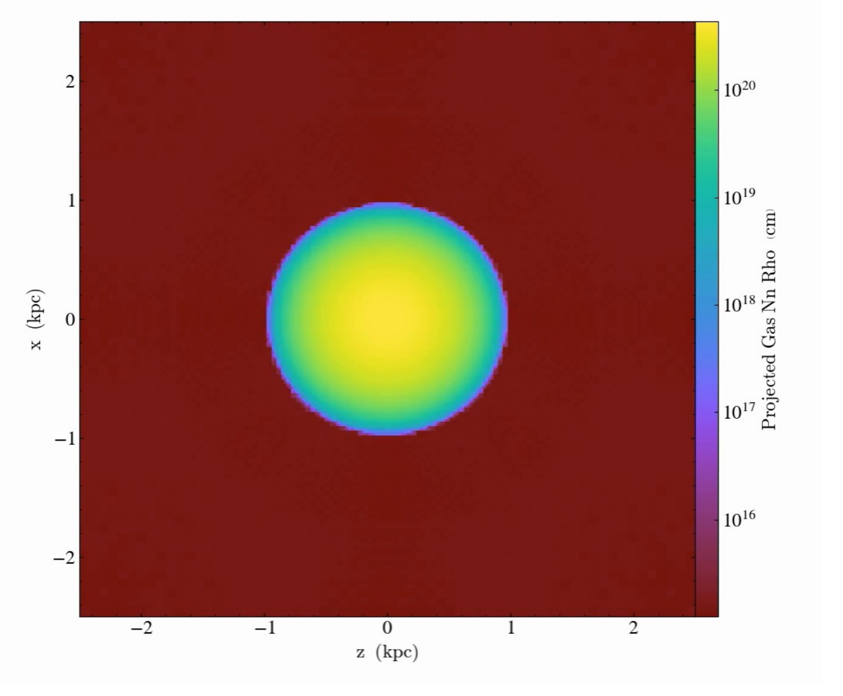
Prust & Chang (2019)

MANGA

Chang, Davis & Jiang (2020)

Chang & Etienne (2020)





Radiation

GR Hydrodynamics
In static spacetimes

MANGA - A Moving Mesh Solver for ChaNGa

Current Features

- Hydrodynamics on Voronoi Mesh, Self-gravity, Entropy or Energy solving (Chang, Quinn & Wadsley 2017)
- Multistepping (Prust & Chang 2019)
- MESA Stellar Equation of State (Prust & Chang 2019)
- Moving and Reactive Boundary Conditions (Prust 2020)
- Radiation Hydrodynamics (Chang, Davis & Jiang 2020)
- GR hydrodynamics on the moving-mesh (Chang & Etienne 2020)

Near-Term Goals (< 2 years)

- Open source version in early-mid 2021
- MHD: constrained transport scheme (Prust & Chang, in prep)
- Moving-mesh GRHD for BNS Mergers

Longer Term Goals (~ 2-4 years)

- High Order Spatial Reconstruction Methods
- Core-collapse SN on a moving-mesh with neutrino radiation
- Point Source Radiation

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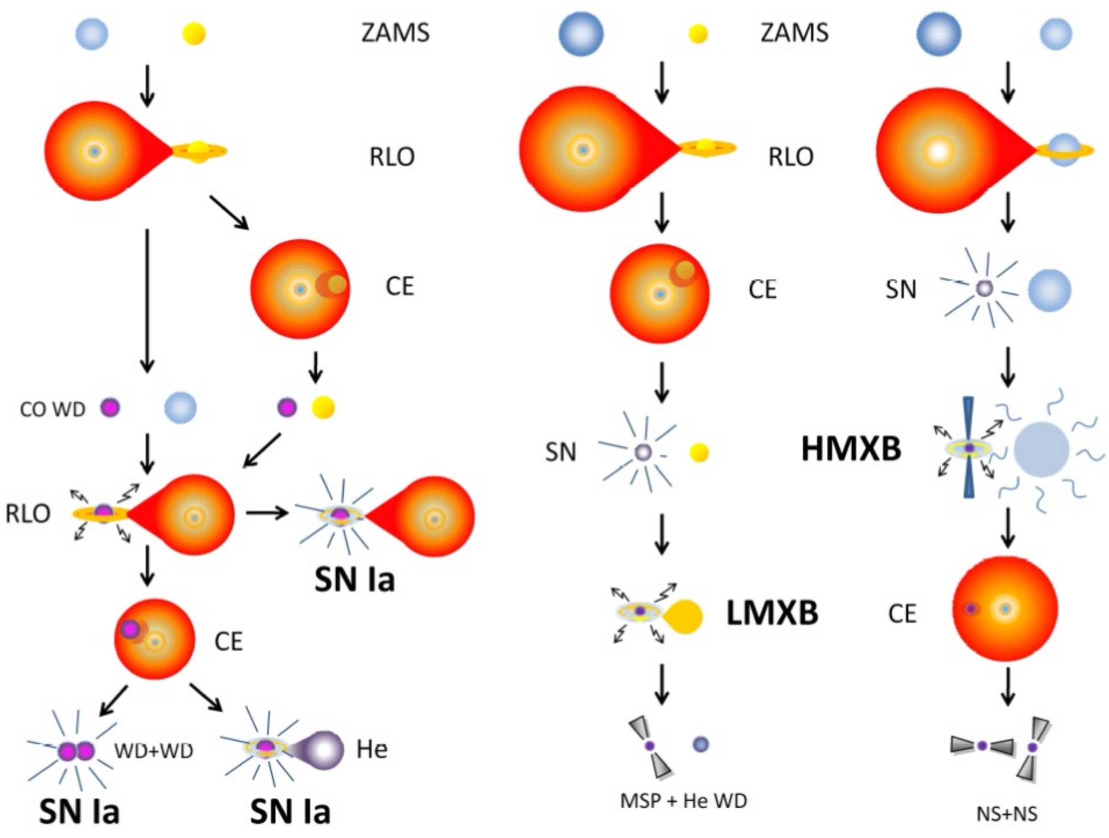
Common Envelope Evolution

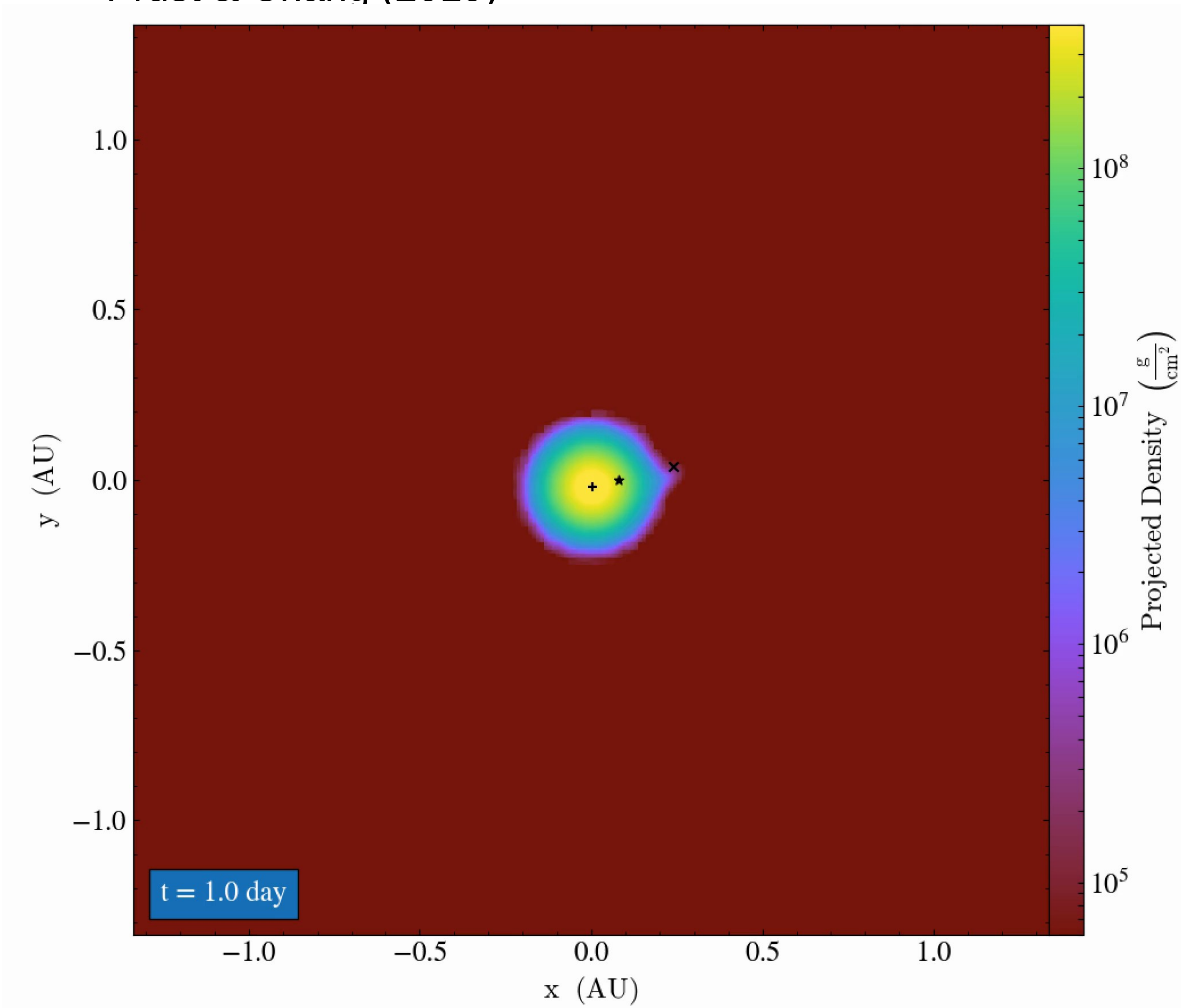
Ivanova et al. (2012)



- In a close binary system, a star that evolves up the RGB/AGB may fill its Roche lobe.
- For unstable mass transfer, the secondary may fall into the primary's envelope – "common envelope"
- The secondary and primary's core spiral in toward each other.
- Release of gravitational potential energy is balanced by ejection of the envelope.
- Results in a close binary pair
  - Possibly responsible for progenitors of:
    - SN Ia
    - millisecond pulsars
    - binary neutron stars
    - binary black holes.

CEE using MANGA

Prust & Chang (2019)



We use similar initial conditions as Ohlmann et al (2016)

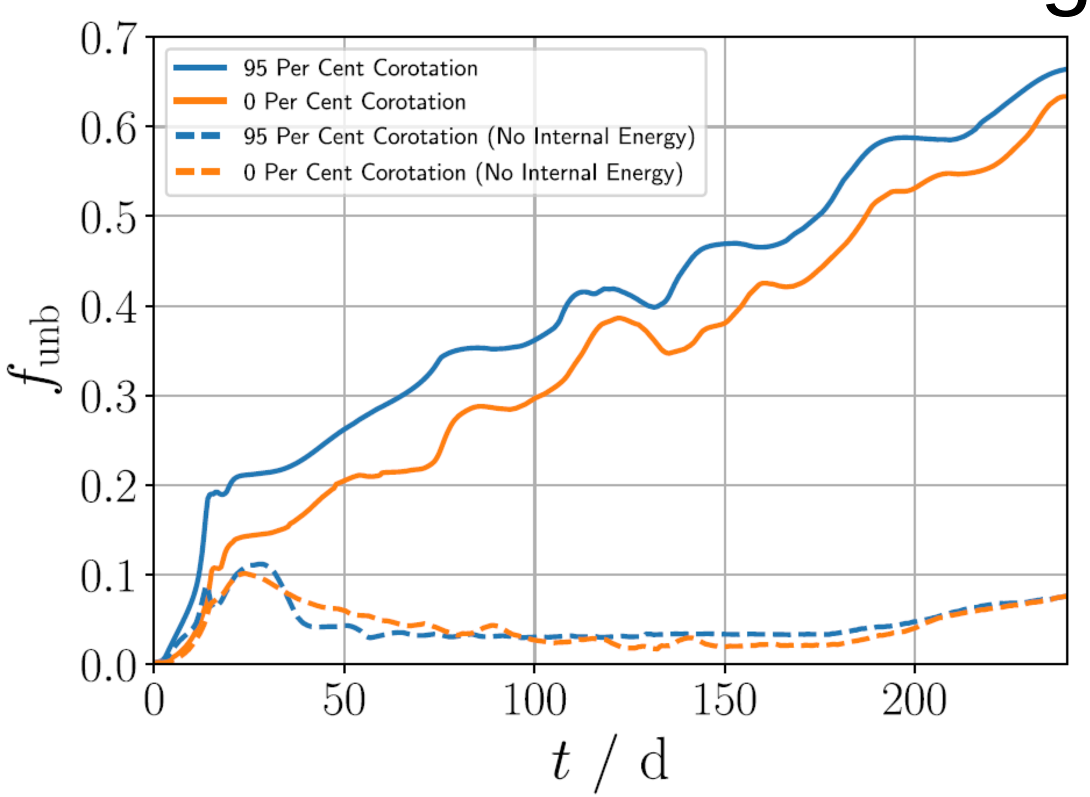
2 solar mass RG at 52 solar radii, 1 solar mass secondary – treated as DM particle.

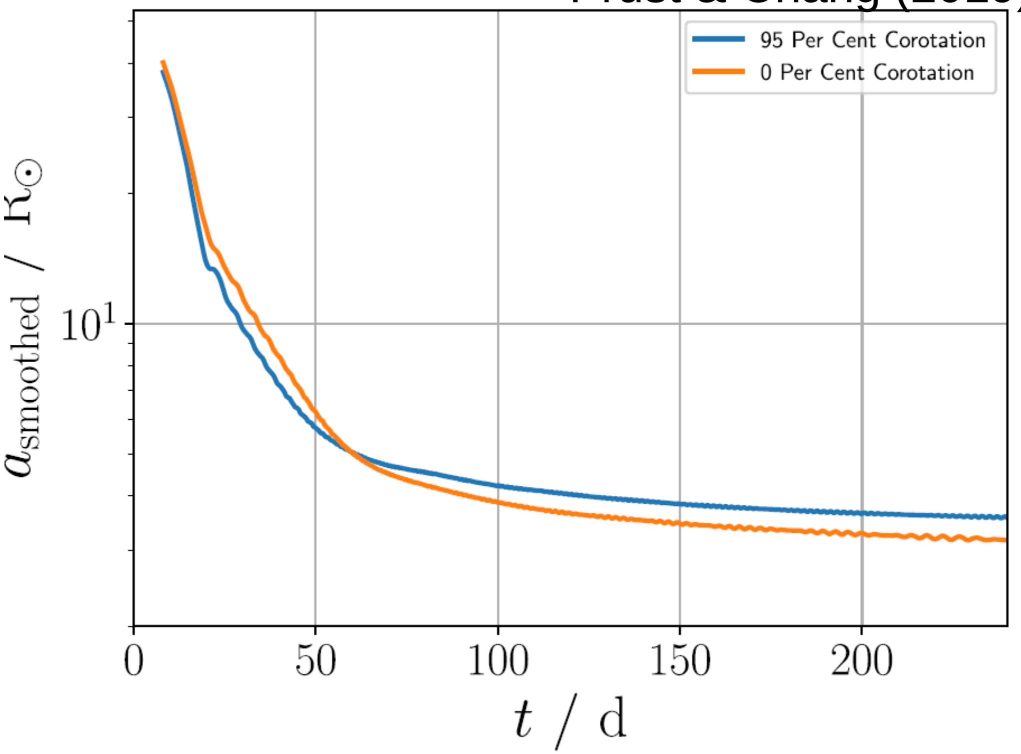
Use about 400K particles to model the RG, 800K particles altogether (including atmospheric particles).

Run for 240 days – 110 shown here.

CEE using MANGA

Prust & Chang (2019)





We find that a substantial amount of envelope can be ejected depending on how you account for the energy of expansion.

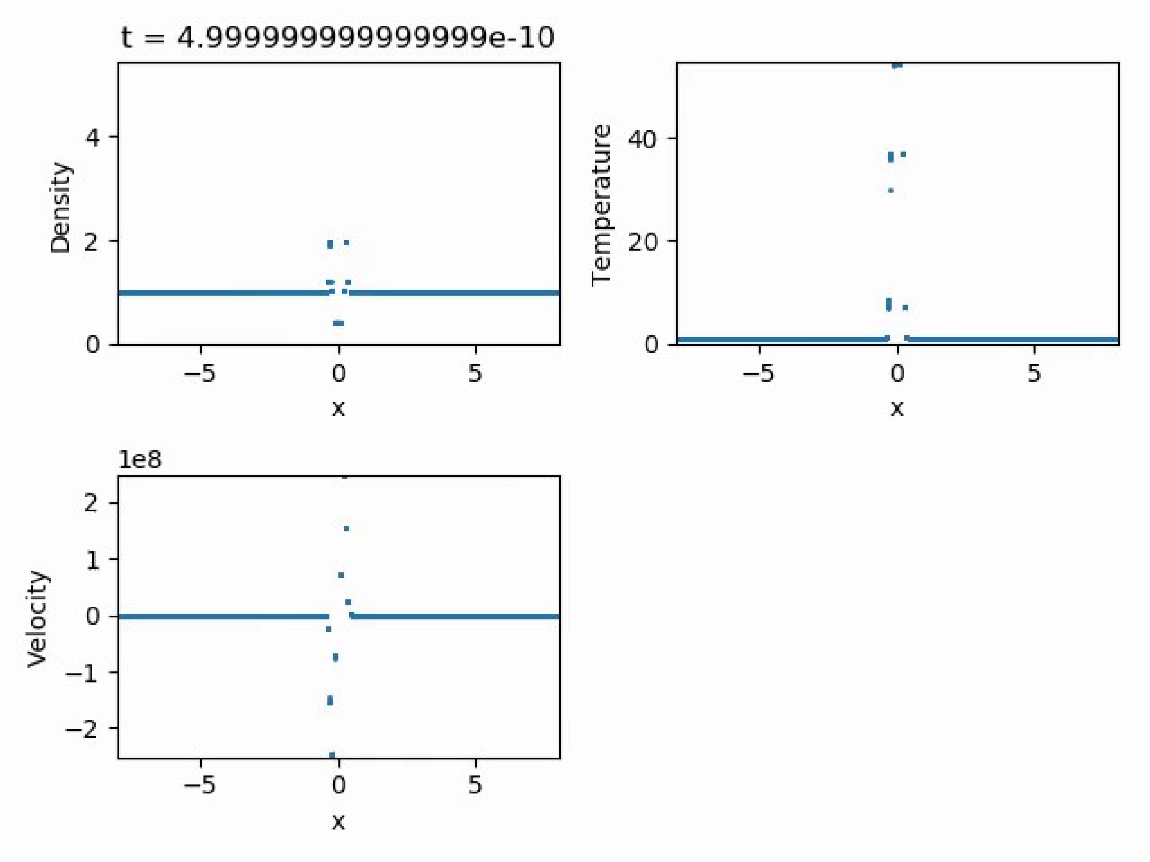
Including thermal energy, we get 66% ejection of the envelope.

Only mechanical energy, we get ~10% ejection – similar to other workers

The orbit shrinks substantially – near the limits of the gravitational softening.

Moving/Reactive Boundary Conditions

- Secondary star is “dense” relative to the envelope – treat it as a moving (reflecting) boundary condition.
- Moving bc must be influenced by the flow – to preserve conservation laws



Prust (2020)

Start/End of Wall

Apply reflecting boundary conditions to certain cells, but account for the forces applied on it.

Linked these boundary cells to move with a common velocity + center

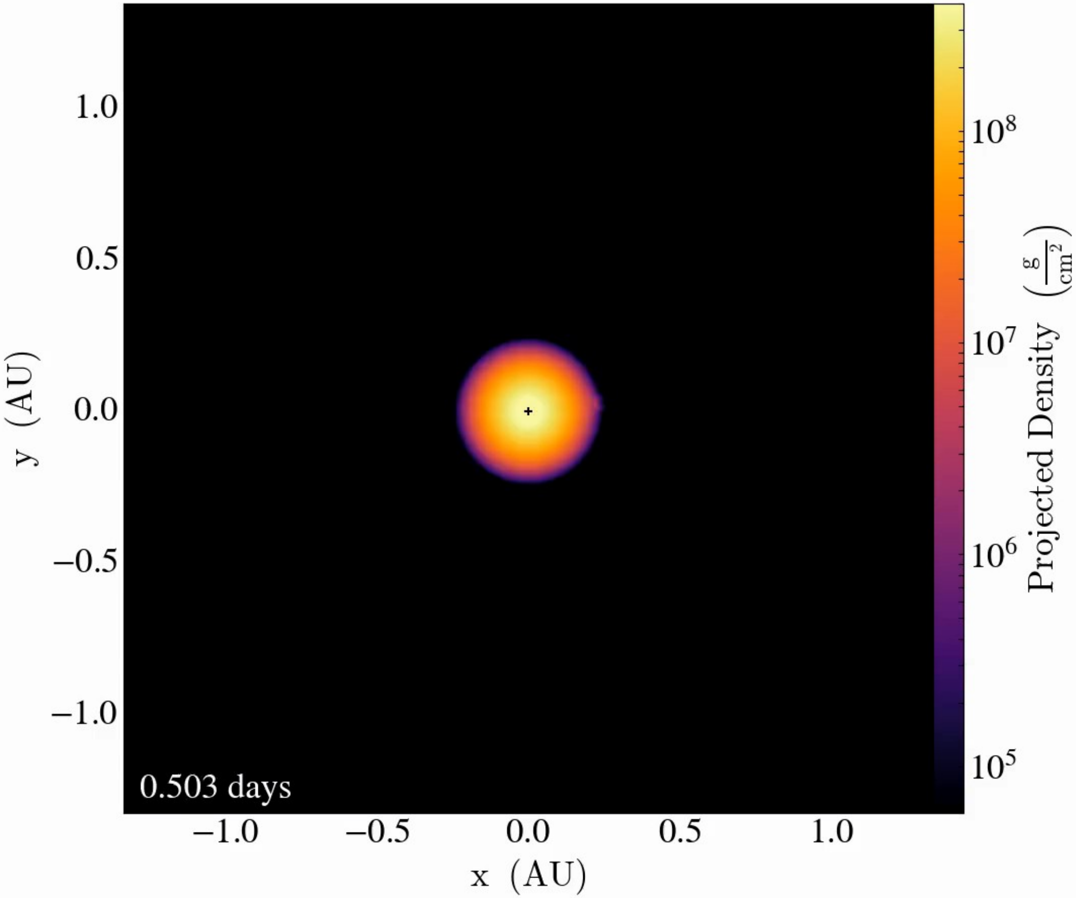
Gas cells immediately neighboring the boundary cells are also locked into their motion.

“1-d” problem of a Sedov shock hitting a piston at x=3 to 5 initially.

Conservation of linear momentum to within a few percent for sufficient resolution.

CEE with a “hard” secondary

Prust (2020)

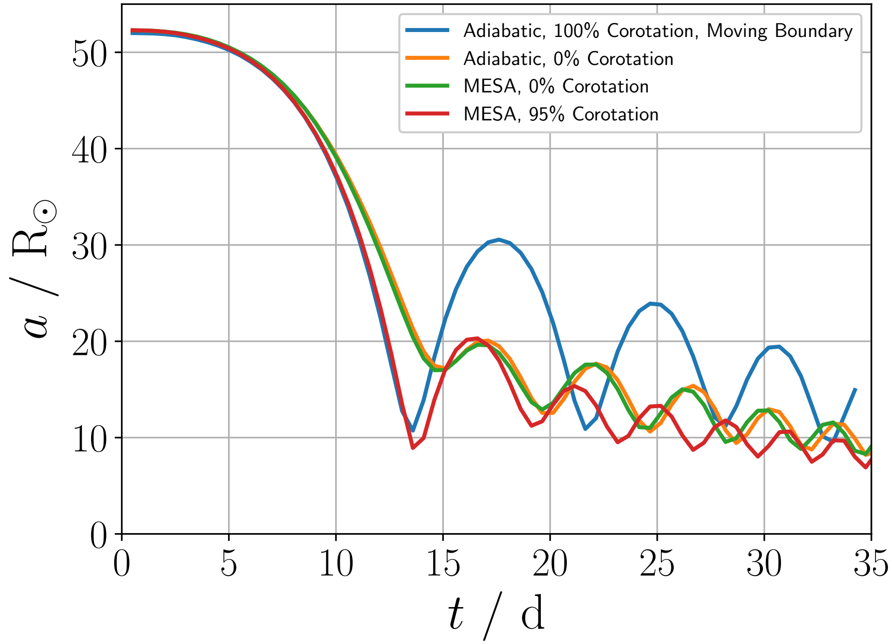


Moving BC run with same initial conditions as Prust & Chang (2019)

Somewhat different inspiral evolution

More analysis remains to be done

Prust (2020)



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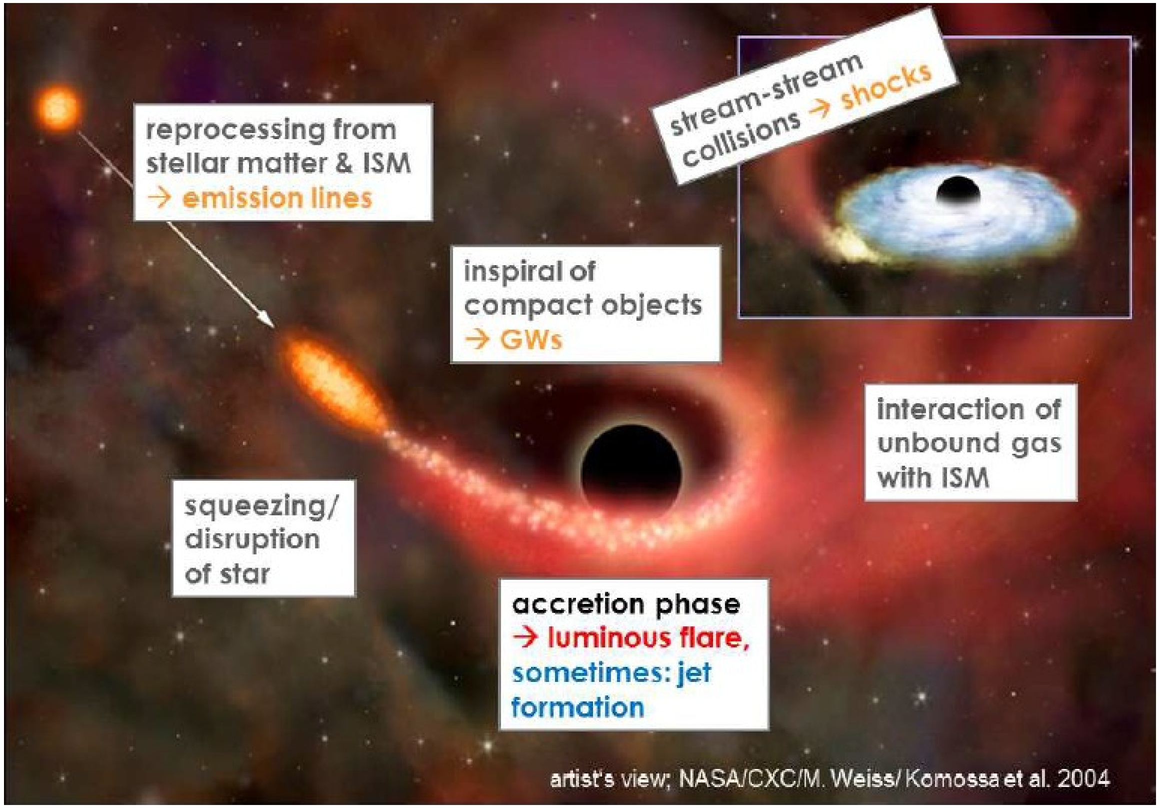
Tidal Disruption Events

Komossa (2015)



A star that falls in close to a SMBH can get ripped apart by tides.

Called a tidal disruption event (TDE)

Half of the star is bound to the BH and will accrete onto the BH on a month-year-decade long timescale.

Accretion rate and luminosity follows a $t^{-5/3}$ power law.

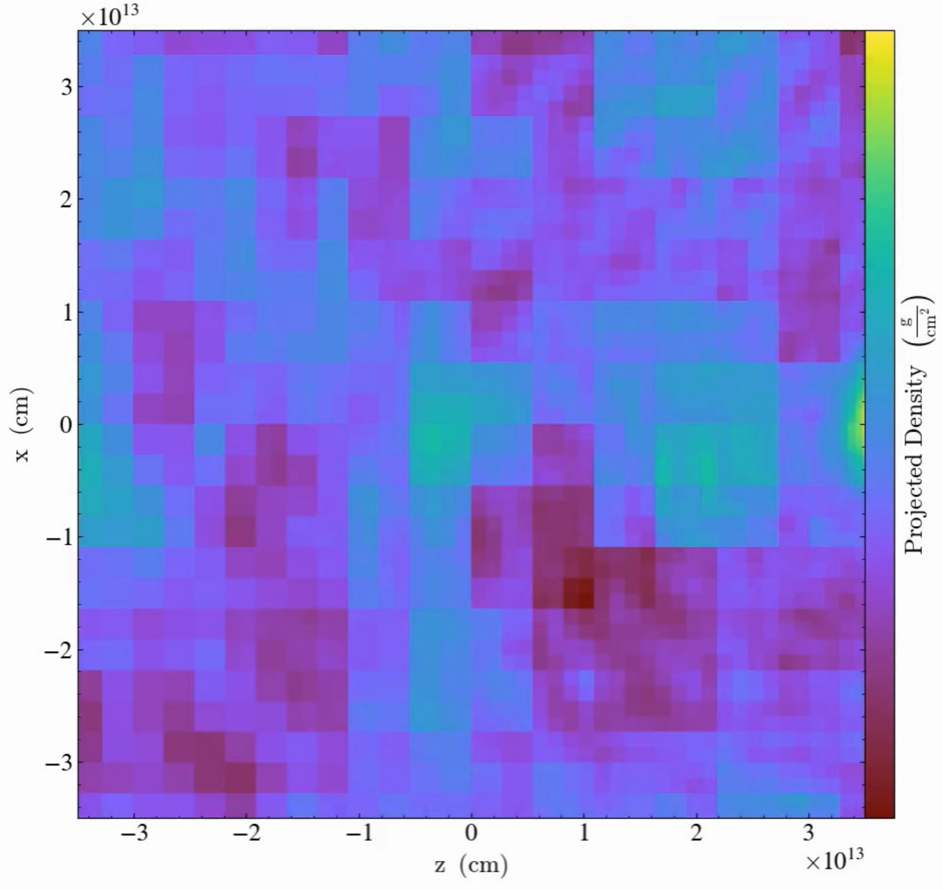
Emission during TDE events occurs in several different phases:

- Initial disruption + shock breakout (Guillochon et al 2009)
- Collision of streams (Jiang et al. 2016)
- Fallback and circularization (Hayasaki et al. 2016)
- Accretion disk
- Reprocessed radiation (Strubbe & Quataert 2011) – emission line transients
- Shocking of unbound gas (Yalinewich et al. 2019) – radio transients

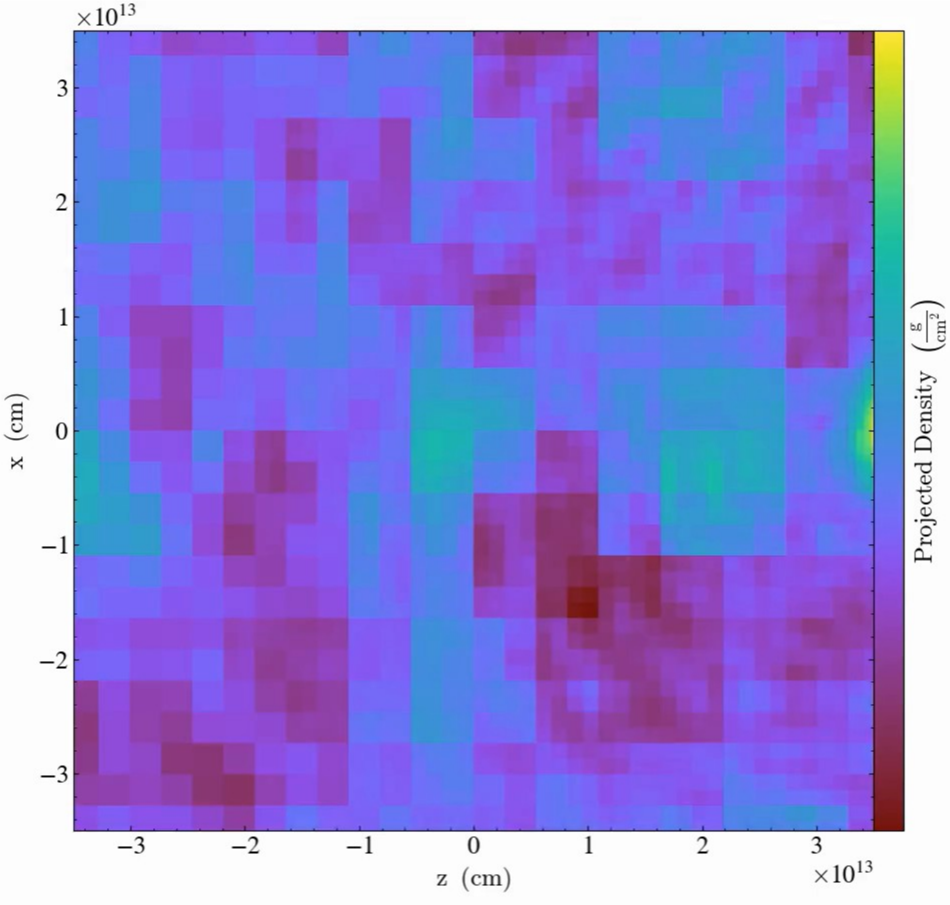
Simulations of Tidal Disruption Events

- Simulations of TDEs were first done with SPH (Evans & Kochanek 1989,
- Simulations of TDEs with Eulerian codes, AMR grid centered on the star (black hole moving by)
- Find $t^{-5/3}$ power law, larger energy distribution – earlier start times for fallback, possible shock breakout during initial disruption, importance of GR for circularization
- Why Moving-mesh?
  - Capture shocks – initial disruption shock
  - Can include additional physics – (diffuse) radiation, magnetic fields
  - Capture the entire domain
- Few simulations already with moving mesh

Tidal Disruption Events

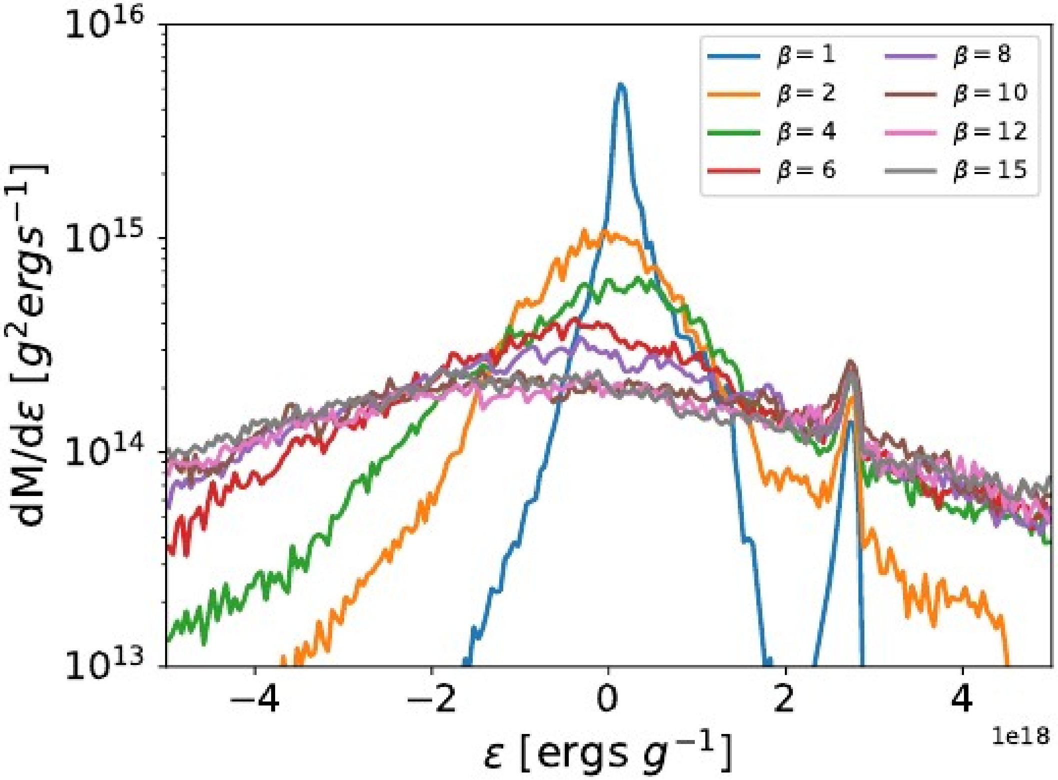


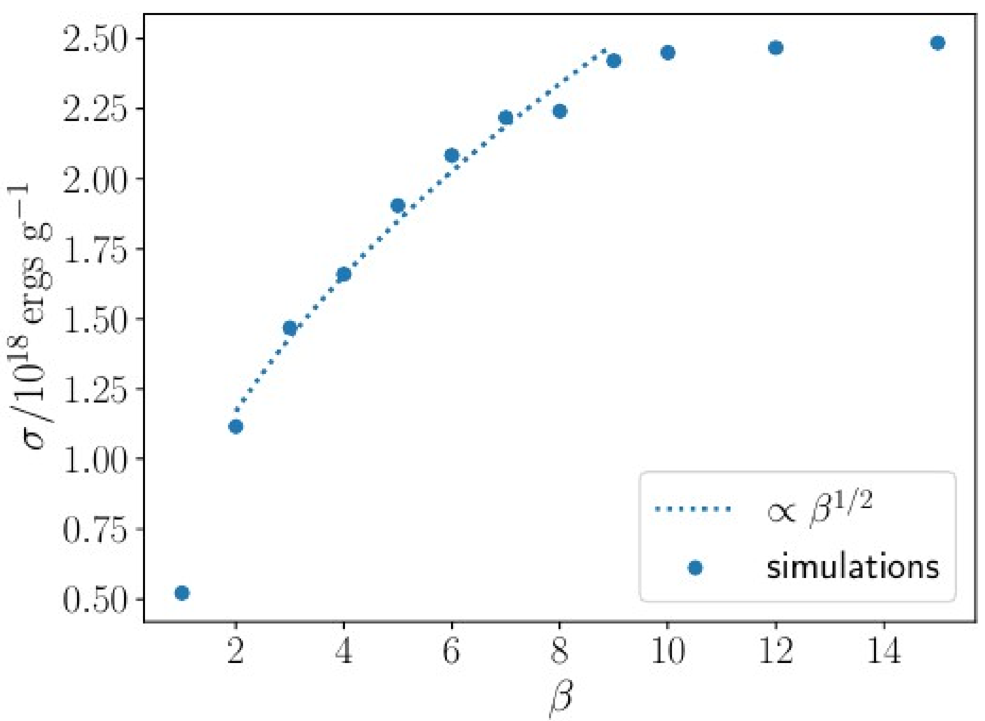
$$\beta \equiv \frac{r_t}{r_p} = 2$$



$$\beta = 4$$

Effect of β





- Spread in energy depends on β < 9.
- Scales like $\beta^{-1/2}$ for β = 2-9, fixed afterwards
- Gives a corresponding decrease in accretion rate

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GRHD on a Moving-mesh

GRHD can also be written as a flux-conservative equation

$$\frac{\partial u}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{\mathcal{F}} = \mathcal{S}$$

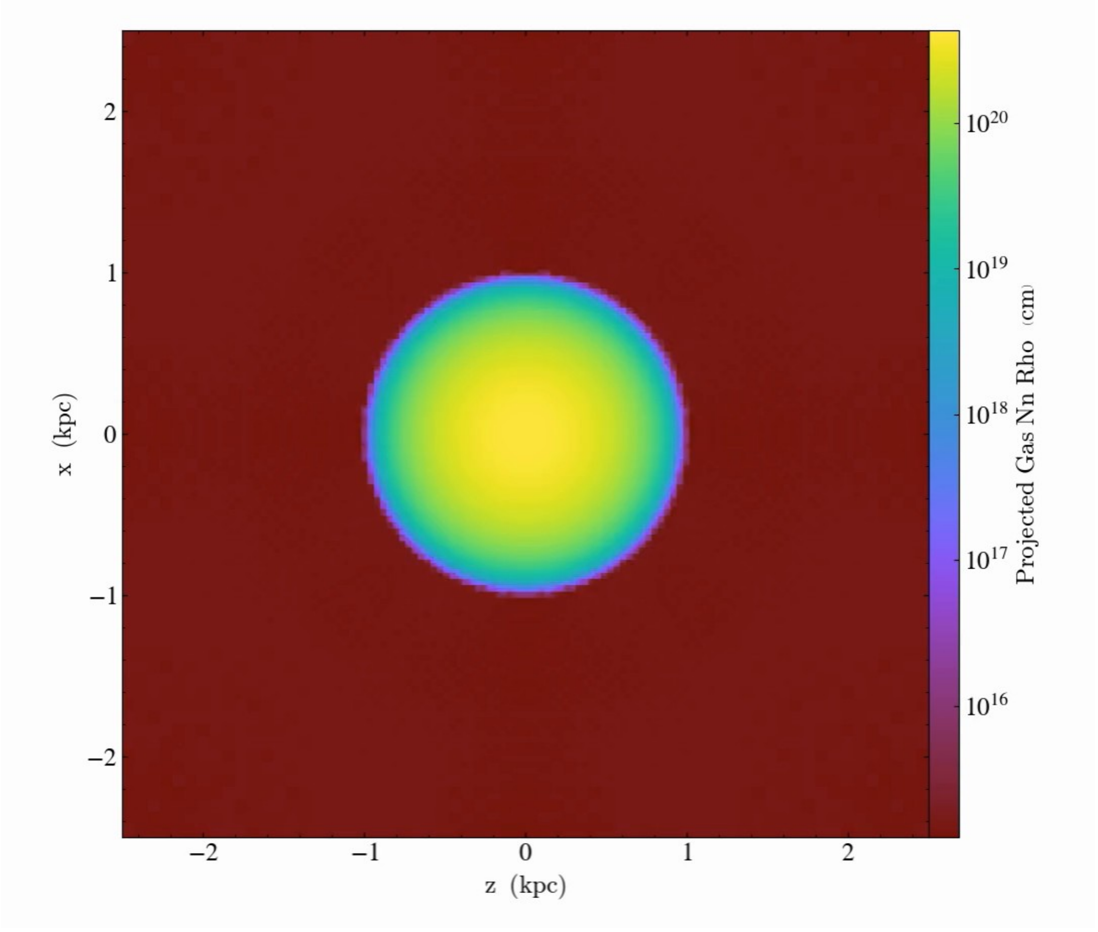
$$u = (\rho, \rho \boldsymbol{v}),\ \boldsymbol{\mathcal{F}} = (\rho \boldsymbol{v}, \rho \boldsymbol{v}\boldsymbol{v} + P\mathcal{I}), \mathcal{S} = (0, -\rho \boldsymbol{\nabla} \Phi)$$

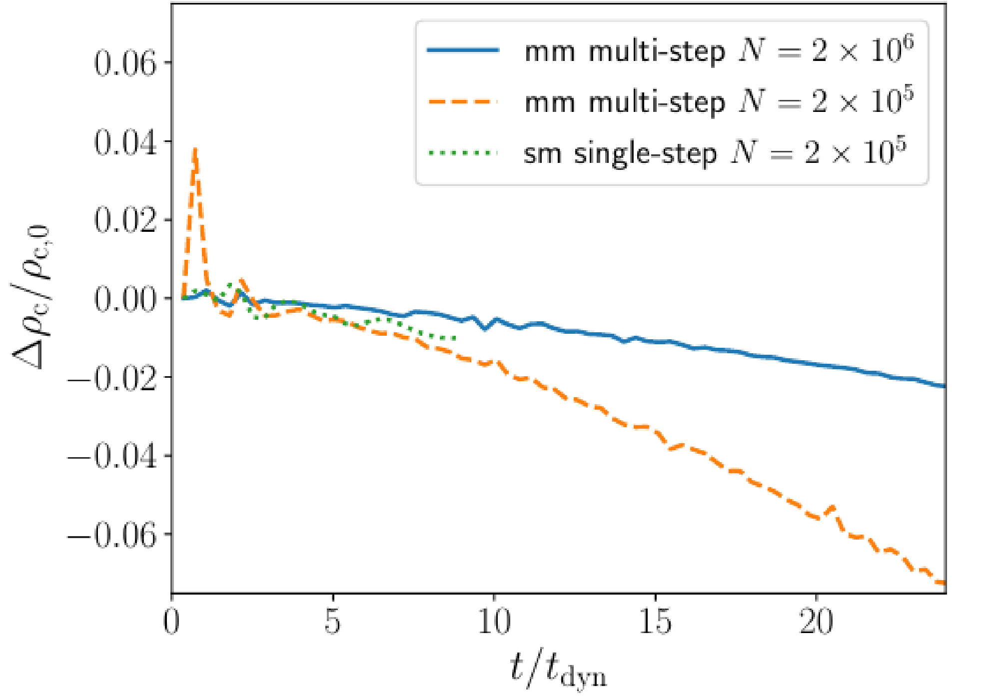
$$u = (\rho_*, \rho_* h \boldsymbol{u}),\ \boldsymbol{\mathcal{F}} = (\rho_* \boldsymbol{v}, \sqrt{-g} T^{j\beta} g_{\beta i}, \mathcal{S} = (0, \frac{\sqrt{-g}}{2} T^{\alpha\beta} g \alpha \beta, i)$$

Where $\rho_* = \sqrt{-g}\rho u^0$, h is the enthalpy

So GRHD can also be solved on a moving unstructured mesh!

TOV star on a Moving-mesh





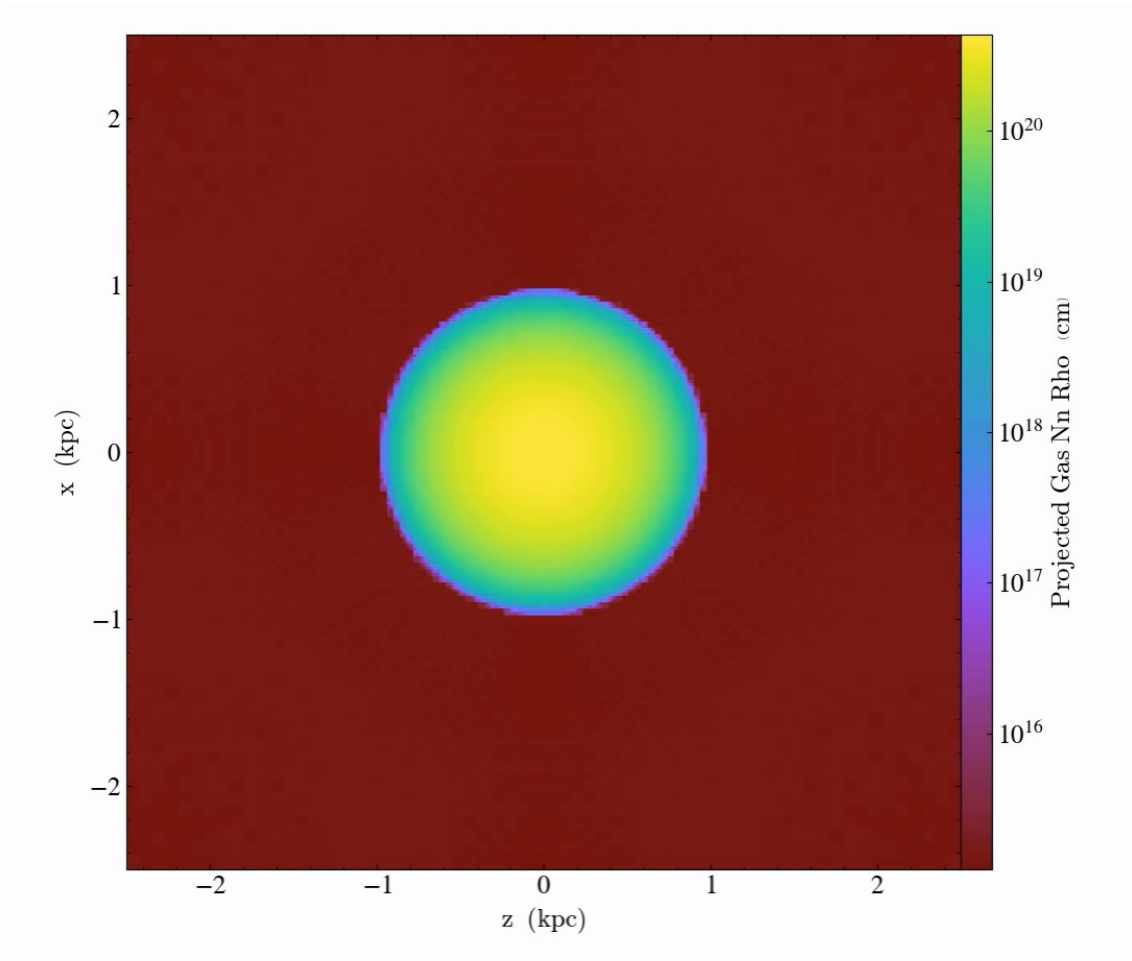
- Star modelled by 10^6 mesh generating points.
- Fixed TOV metric. Run for 24 dynamical times.
- Diffusion of material due to sharp gradient in outer boundary of star

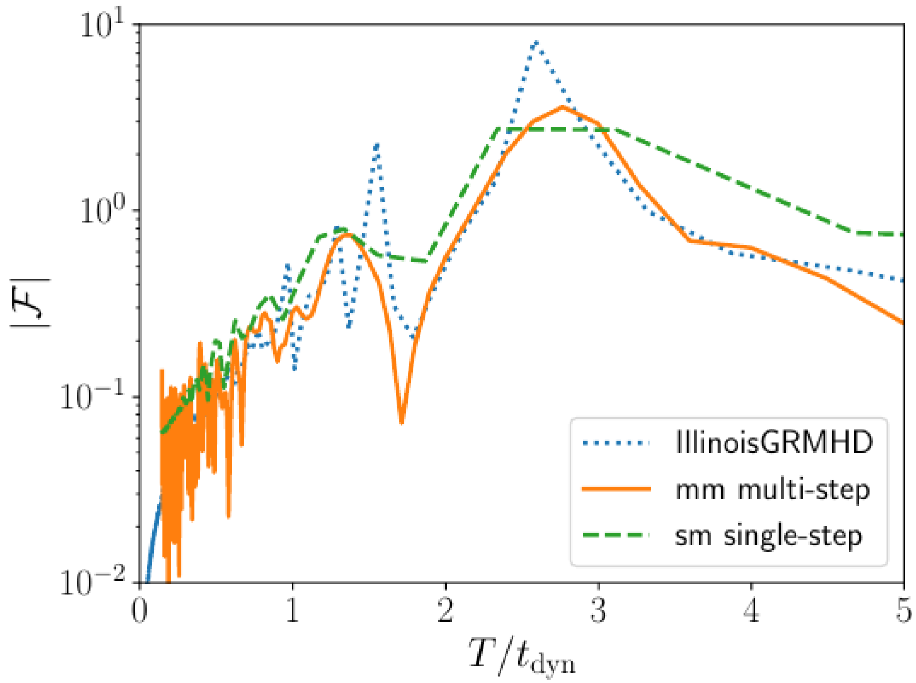
At "high" resolution, secular drift of central density of 2% over 24 dynamical times.

Single star evolutions is really sensitive to spatial reconstruction (Duez et al 2005)

May be fixed in near-term with developments in unstructured high-order methods.

TOV star on a Moving-mesh





- Reduce pressure by 10% globally
- Star oscillates radially at the fundamental mode.
- Loss of mass and energy across the sharp gradient at the edge of the star.

Oscillations match those generated by IllinoisGRMHD for same initial conditions

Future is incorporating a dynamical spacetime solver into MANGA for full moving-mesh BNS mergers simulations.

Conclusions

- Moving-mesh schemes have a number of advantages (and disadvantages) for astrophysics.
  - A number of open source codes (AREPO, MANGA) will be available soonish

- Particularly well suited to a number of dynamical stellar problems
  - Common Envelope Evolution (shocks, moving boundaries, magnetic fields, radiation)
  - Tidal Disruption Events (shocks, $v >> c_s$, magnetic fields, radiation)

- We have found that envelope ejection is possible provide a means to “tap” thermal energy
  - Require radiative transfer to do this correctly
  - Moving/reactive BC work ongoing

- We have found that energy distribution and mass accretion rate depends on impact parameter – possible means to constrain impact parameter

- GR Hydrodynamics with static spacetimes is now working; dynamical spacetimes are next.

- We anticipate open-source version available sometime in 2021

Radiation Hydro on a Moving-mesh

Euler equation among others can be written as a flux-conservative equation

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Can be solve in a finite volume scheme

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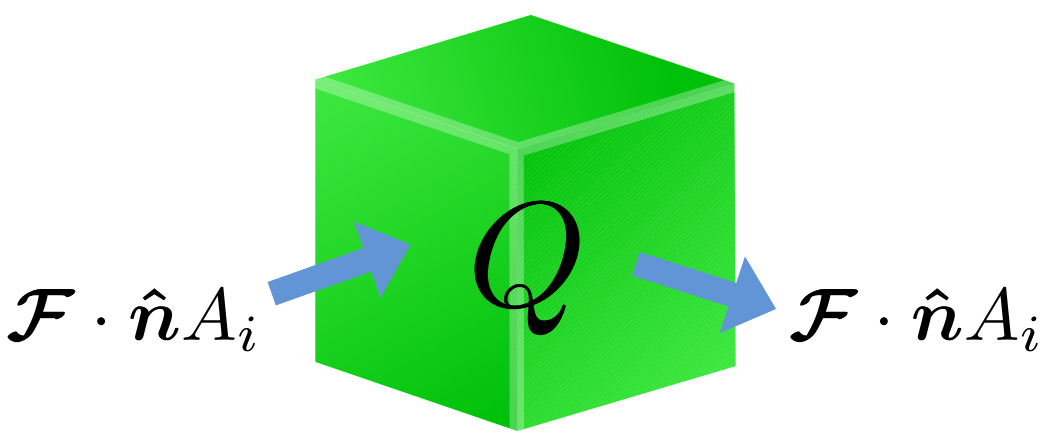

Fluxes are solved with a (approximate) Riemann solver

MANGA vs AREPO

AREPO

- Based on Gadget 3 SPH/N-body code
- Voronoi tessellation based on computing dual to Delauncy tessellation.
  - Can do 1-, 2-, 3-d calculations
- Gradient estimate using least-squares fitting.
- Second order scheme needs 1 voronoi construction, 2 Riemann solves.
- Used mainly for cosmology/galaxies
- Is now “open source”.

MANGA

- Based on ChaNGa SPH/N-body code
  - Successor to Gasoline
- Directly computes Voronoi tessellation using VORO++ library (Rycroft 2009)
  - Only 3-d calculations
- Gradient estimation based on center of mass coordinates of cell.
- Second order scheme needs 2 voronoi constructions, 1 Riemann solve.
- Used mainly for dynamical stellar problems
- Planned “open source” – 1H 2020