Histogram sort with Sampling (HSS)

Vipul Harsh, Laxmikant Kale

Parallel sorting in the age of Exascale

- Charm N-body GrAvity solver
- Massive Cosmological N-body simulations
- Parallel sorting in every iteration

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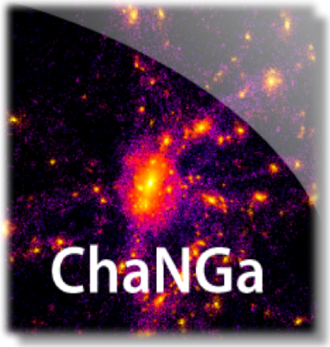

- Charm N-body GrAvity solver
- Massive Cosmological N-body simulations
- Parallel sorting in every iteration

- Cosmology code based on Chombo
- Global sorting every step for load balance/locality

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Parallel sorting: Goals

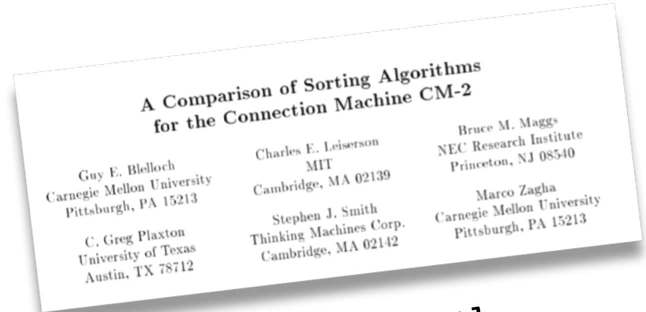
- Load balance across processors
- Optimal data movement
- Generality: robustness to input distributions, duplicates
- Scalability and performance

Parallel sorting: A basic template

- *p* processors, N/*p* keys in each processor
- Determine (*p*-1) splitter keys to partition keys into *p* buckets
- Send all keys to appropriate destination bucket processor
- Eg. Sample sort, Histogram sort

Existing algorithms: Parallel Sample sort

- Samples *s* keys from each processor
- Picks (p-1) splitters from p x s samples


A Comparison of Sorting Algorithms
for the Connection Machine CM-2

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[SPAA' 91]

Problem: Too many samples required for good load balance

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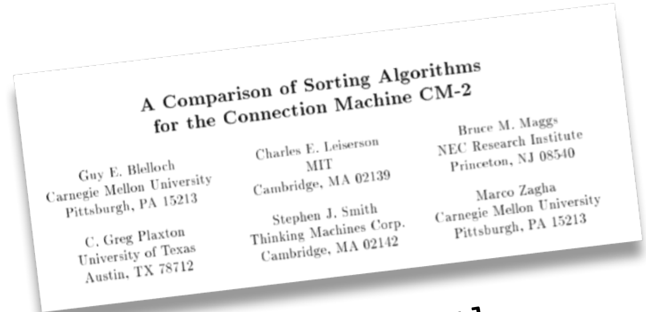

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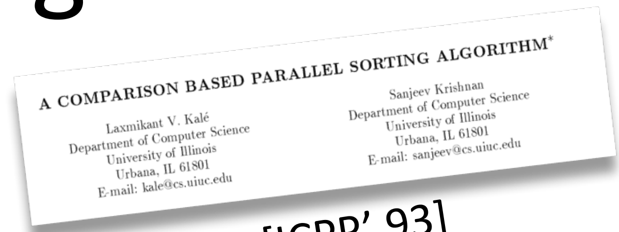
Problem: Too many samples required for good load balance

64 bit keys, p = 100,000 & 5% max load imbalance, sample size ≈ 8 GB

Existing algorithms: Histogram sort


A COMPARISON BASED PARALLEL SORTING ALGORITHM*

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[ICPP' 93]

- Pick *s x p* candidate keys
- Compute rank of each candidate key (histogram)
- Select splitters from the candidates

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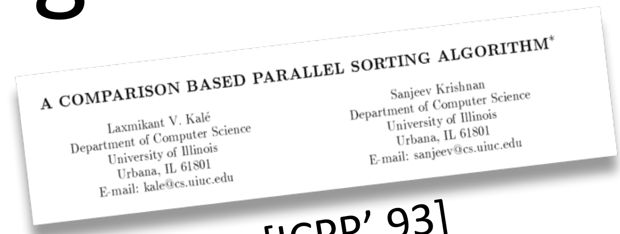
Existing algorithms: Histogram sort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- Refine the candidates and repeat

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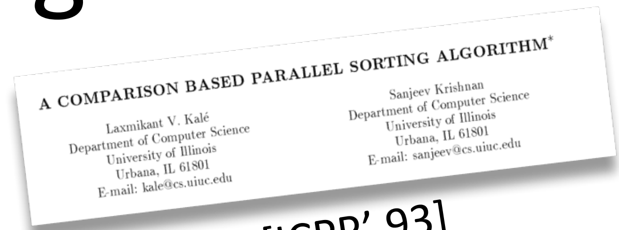
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[ICPP' 93]

- Pick *s x p* candidate keys
- Compute rank of each candidate key (histogram)
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OR

- Refine the candidates and repeat

- Works quite well for large *p*
- But can take more iterations if input skewed

Histogram sort with sampling (HSS)

- An adaptation of Histogram sort
- Sample before each histogramming round
    - Sample intelligently
    - Use results from previous rounds
    - Discard wasteful samples at source

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- HSS has sound theoretical guarantees
- Independent of input distribution
- Justifies why Histogram sort does well

HSS: Intelligent Sampling

Find (p-1) splitter keys to partition input into p ranges

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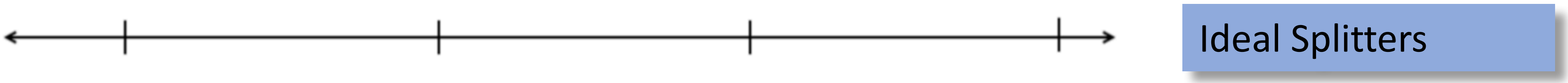

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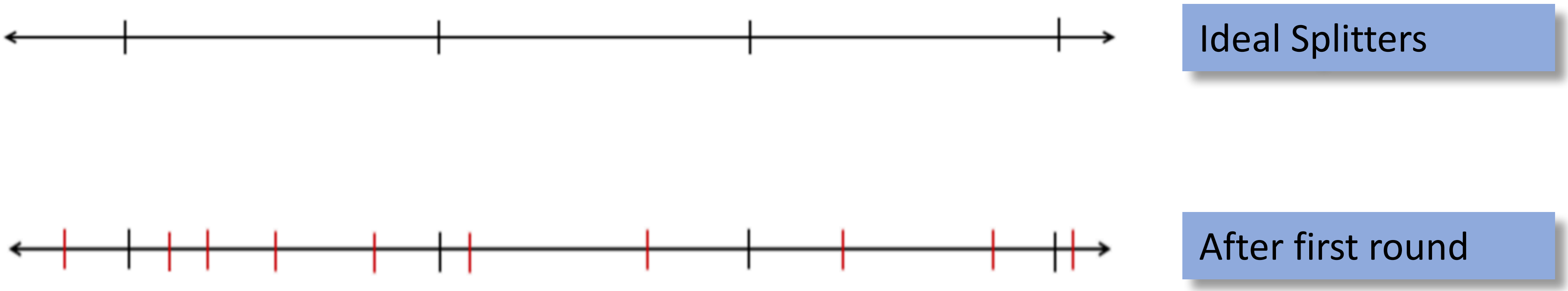

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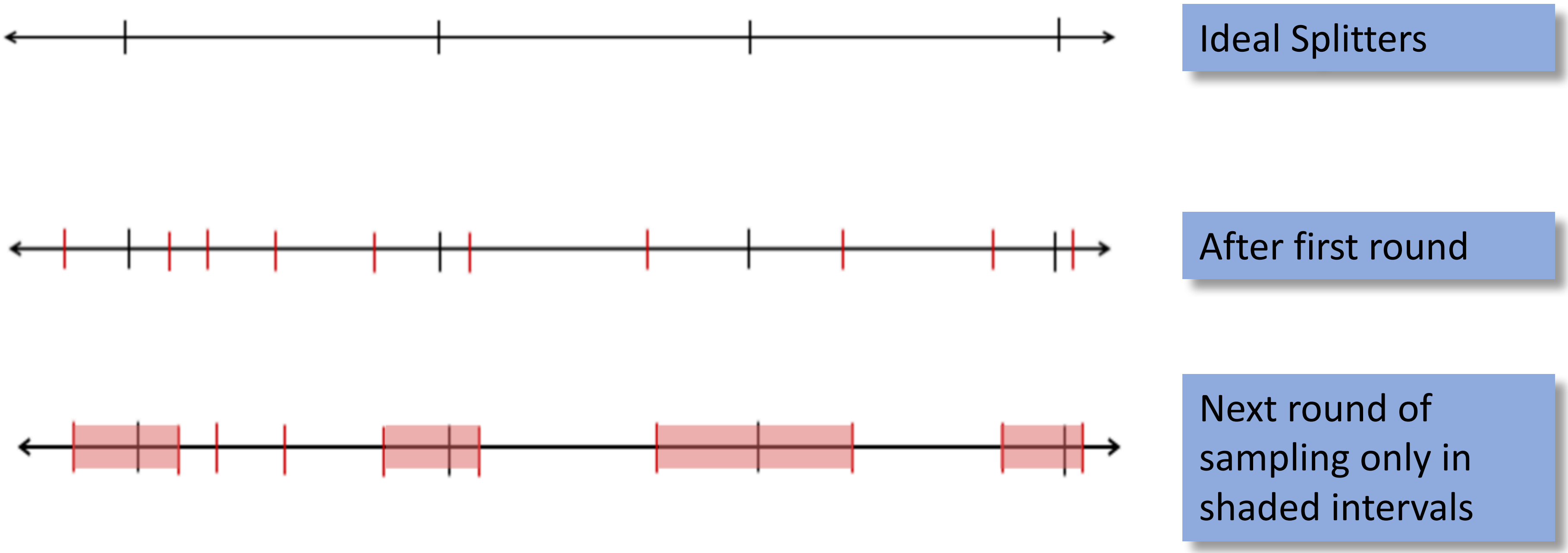

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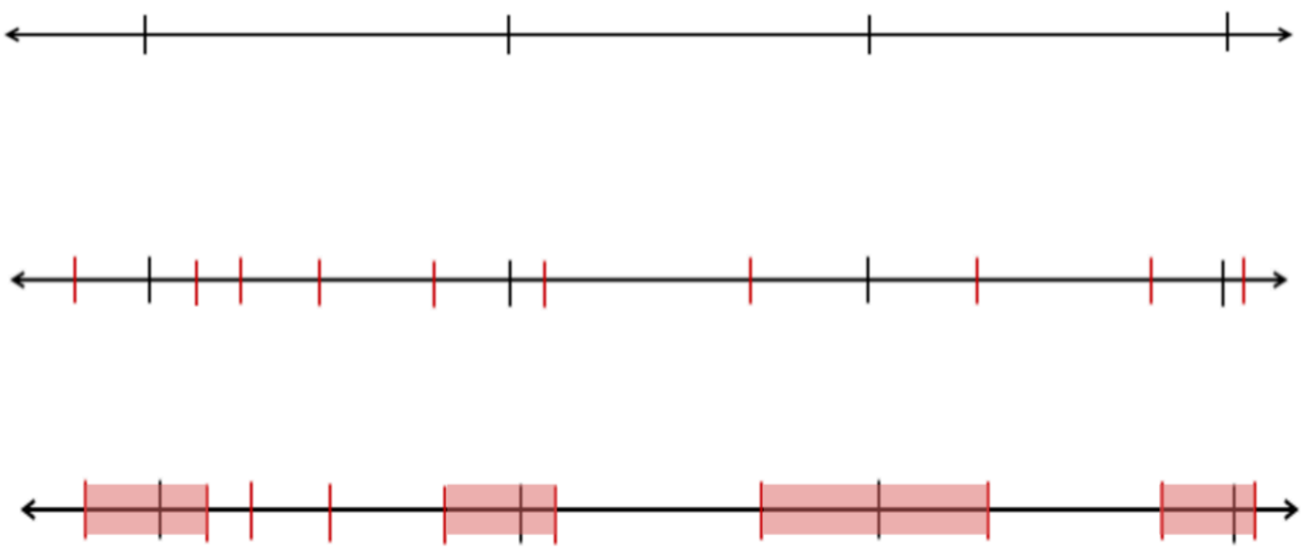

Ideal Splitters

After first round

Next round of sampling only in shaded intervals

Samples outside the shaded intervals are wasteful

HSS: Sample size

Algorithm	Overall sample size	Overall sample size for $p = 10^5, \epsilon = 5\%$
Sample sort with regular sampling	$\mathcal{O}(\frac{p^2}{\epsilon})$	1600 GB
Sample sort with random sampling	$\mathcal{O}(\frac{p \log N}{\epsilon^2})$	8.1 GB
HSS with one round	$\mathcal{O}(\frac{p \log p}{\epsilon})$	184 MB
HSS with two rounds	$\mathcal{O}(p\sqrt{\frac{\log p}{\epsilon}})$	24 MB
HSS with k rounds	$\mathcal{O}(kp\sqrt[k]{\frac{\log p}{\epsilon}})$	-
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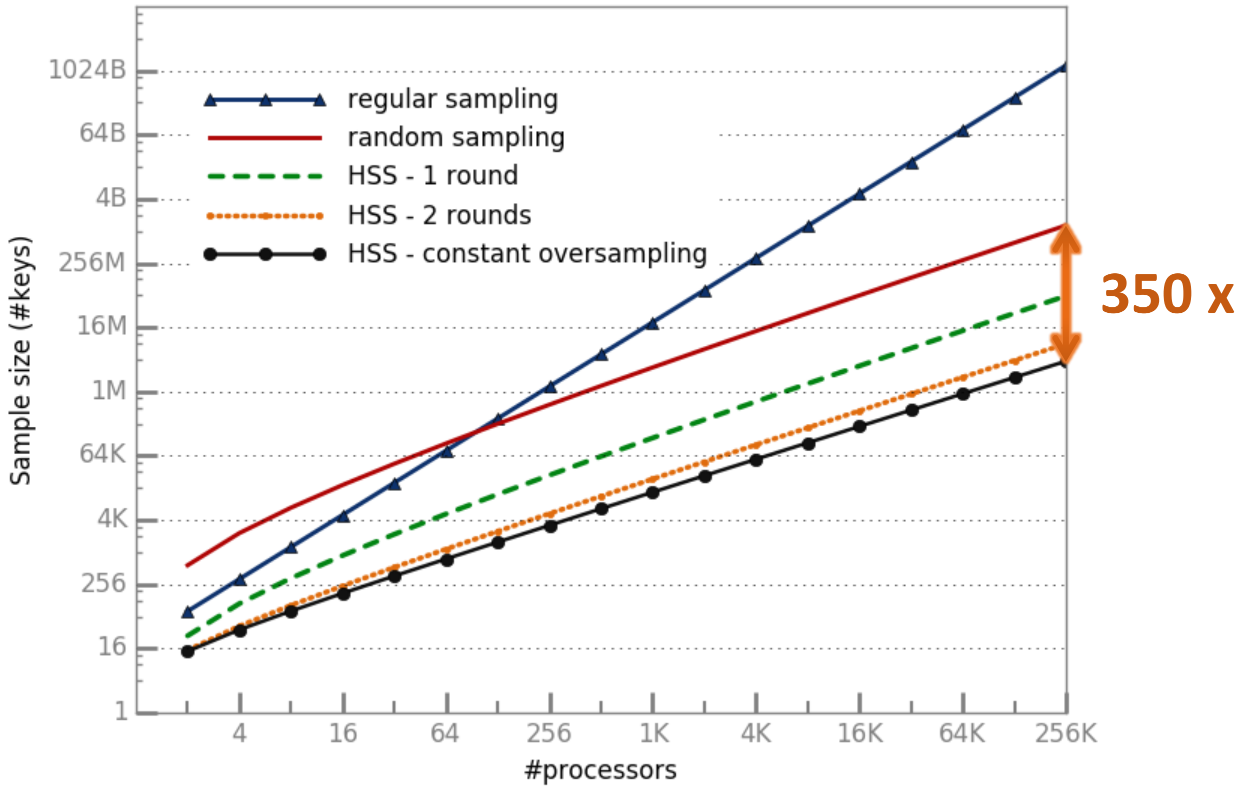

64 bit keys, 5% load imbalance

Number of histogram rounds

p (x 1000)	sample size/round (x p)	Number of rounds	Number of rounds (Theoretical)
4	5	4	8
8	5	4	8
16	5	4	8
32	5	4	8

Number of rounds hardly increases with p → log (log p) complexity

Optimizing for shared memory

- Modern machines are highly multicore
  - BG/Q: 64 hardware threads/node
  - Stampede KNL(2.0): 272 hardware threads/node

- How to take advantage of within-node parallelism?

Final All-to-all data exchange

- In the final step, each processor sends a data message to every other processor

- O(p^2) fine grained messages in the network

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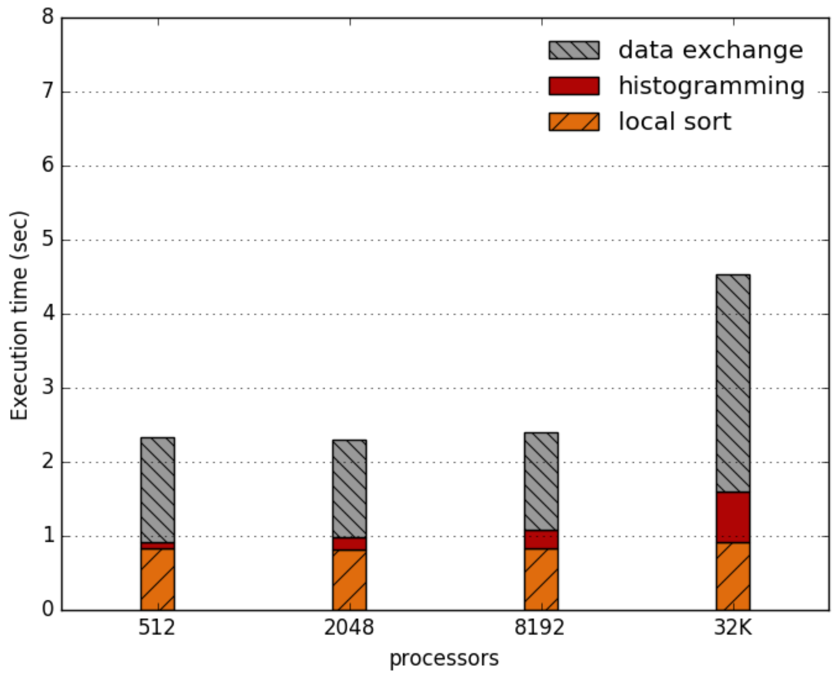
- What if all messages having the same source, destination node are combined into one?

- Messages in the network: O(n^2)
    - Two orders of magnitude less!

What about splitting?…

- We really need splitting across nodes rather than individual processors
- (n-1) splitters needed instead of (p-1)
  - An order of magnitude less
  - Reduces sample size even more
- Add a final within node sorting step to the algorithm

Execution time breakdown



Very little time is spent on histogramming!

Weak Scaling experiments on BG/Q Mira with 1 million 8 byte keys and 4 byte payload per key on each processor, with 4 ranks/node

Conclusion

- HSS combines sampling and histogramming to accomplish fast splitter determination
- HSS provides sound theoretical guarantees
- Most of the running time spent in local sorting & data exchange (unavoidable)

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Future work

- Integration in HPC applications (e.g. ChaNGa)

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Acknowledgements

- Edgar Solomnik
- Omkar Thakoor
- ALCF

Thank You!

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Backup slides

HSS: Computation/Communication complexity

Algorithm	Overall sample size	Overall sample size for $p = 10^5, \epsilon = 5\%$	Computation complexity	Communication complexity
Sample sort with regular sampling	$\mathcal{O}(\frac{p^2}{\epsilon})$	1600 GB	$\mathcal{O}\left(\frac{N}{p}\log\frac{N}{p} + \frac{p^2}{\epsilon}\log p + \frac{N}{p}\log p\right)$	$\mathcal{O}\left(\frac{p^2}{\epsilon} + p + \frac{N}{p}\right)$
Sample sort with random sampling	$\mathcal{O}(\frac{p\log N}{\epsilon^2})$	8.1 GB	$\mathcal{O}\left(\frac{N}{p}\log\frac{N}{p} + \frac{p\log N\log p}{\epsilon^2} + \frac{N}{p}\log p\right)$	$\mathcal{O}\left(\frac{p\log N}{\epsilon^2} + p + \frac{N}{p}\right)$
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HSS with two rounds	$\mathcal{O}(p\sqrt{\frac{\log p}{\epsilon}})$	24 MB	$\mathcal{O}\left(\frac{N}{p}\log\frac{N}{p} + p\sqrt{\frac{\log p}{\epsilon}}\log N + \frac{N}{p}\log p\right)$	$\mathcal{O}\left(p\sqrt{\frac{\log p}{\epsilon}} + p + \frac{N}{p}\right)$
HSS with k rounds	$\mathcal{O}(kp\sqrt[k]{\frac{\log p}{\epsilon}})$	-	$\mathcal{O}\left(\frac{N}{p}\log\frac{N}{p} + kp\sqrt[k]{\frac{\log p}{\epsilon}}\log N + \frac{N}{p}\log p\right)$	$\mathcal{O}\left(kp\sqrt[k]{\frac{\log p}{\epsilon}} + p + \frac{N}{p}\right)$
HSS with $\mathcal{O}(\log\frac{\log p}{\epsilon})$ rounds	$\mathcal{O}(p\log\frac{\log p}{\epsilon})$	10 MB	$\mathcal{O}\left(\frac{N}{p}\log\frac{N}{p} + p\log\frac{\log p}{\epsilon}\log N + \frac{N}{p}\log p\right)$	$\mathcal{O}\left(p\log\frac{\log p}{\epsilon} + p + \frac{N}{p}\right)$

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